Measurement of Dynamic Efficiency: A Parametric Directional Distance Function Approach

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Introduction

- The economics literature on efficiency has traditionally focused on static technical efficiency measures.

- Most advances in the literature on dynamic efficiency modeling have taken place in the nonparametric the DEA framework

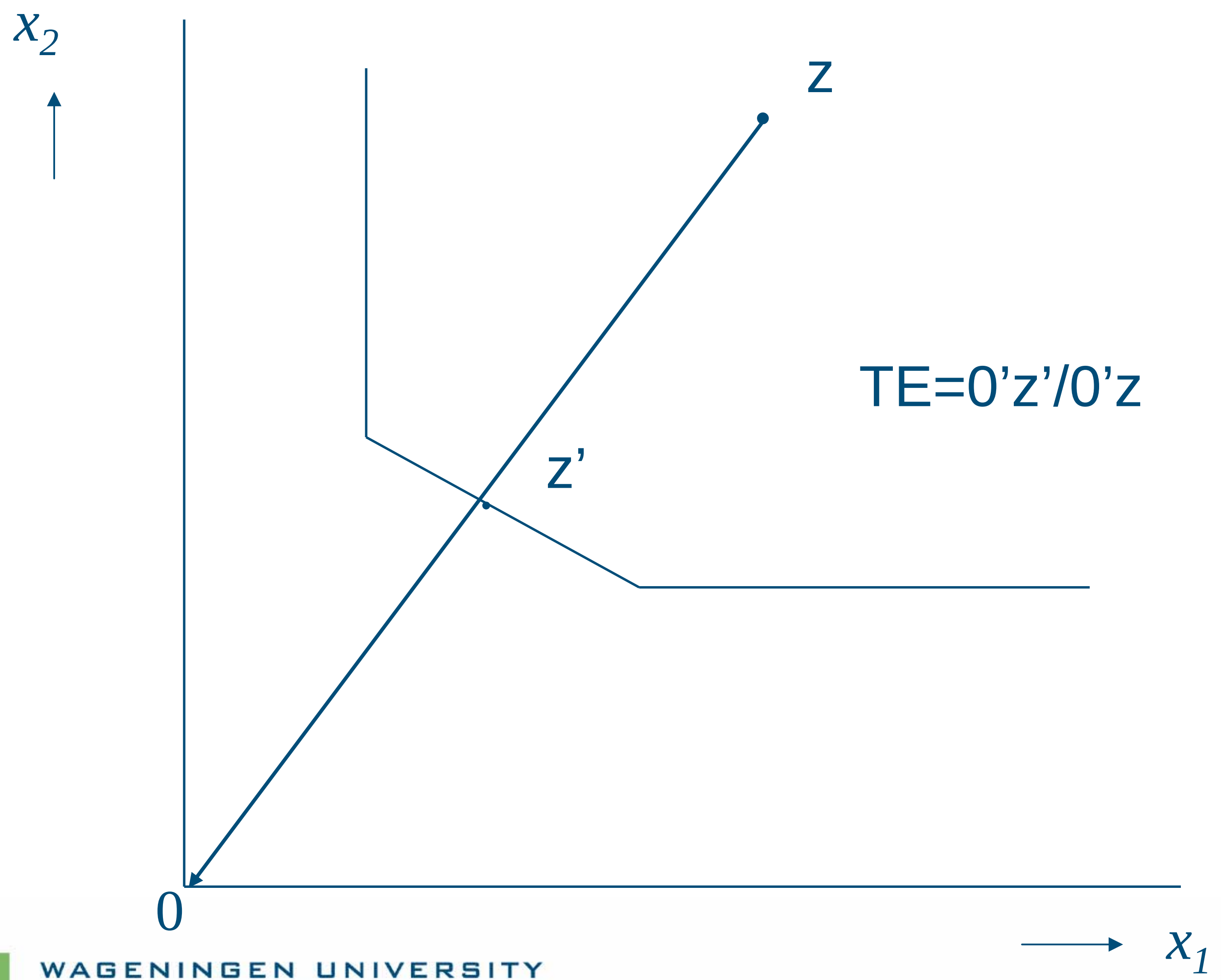
- More recently, the adjustment cost framework has been extended to the directional distance function and its dual cost function.

Introduction

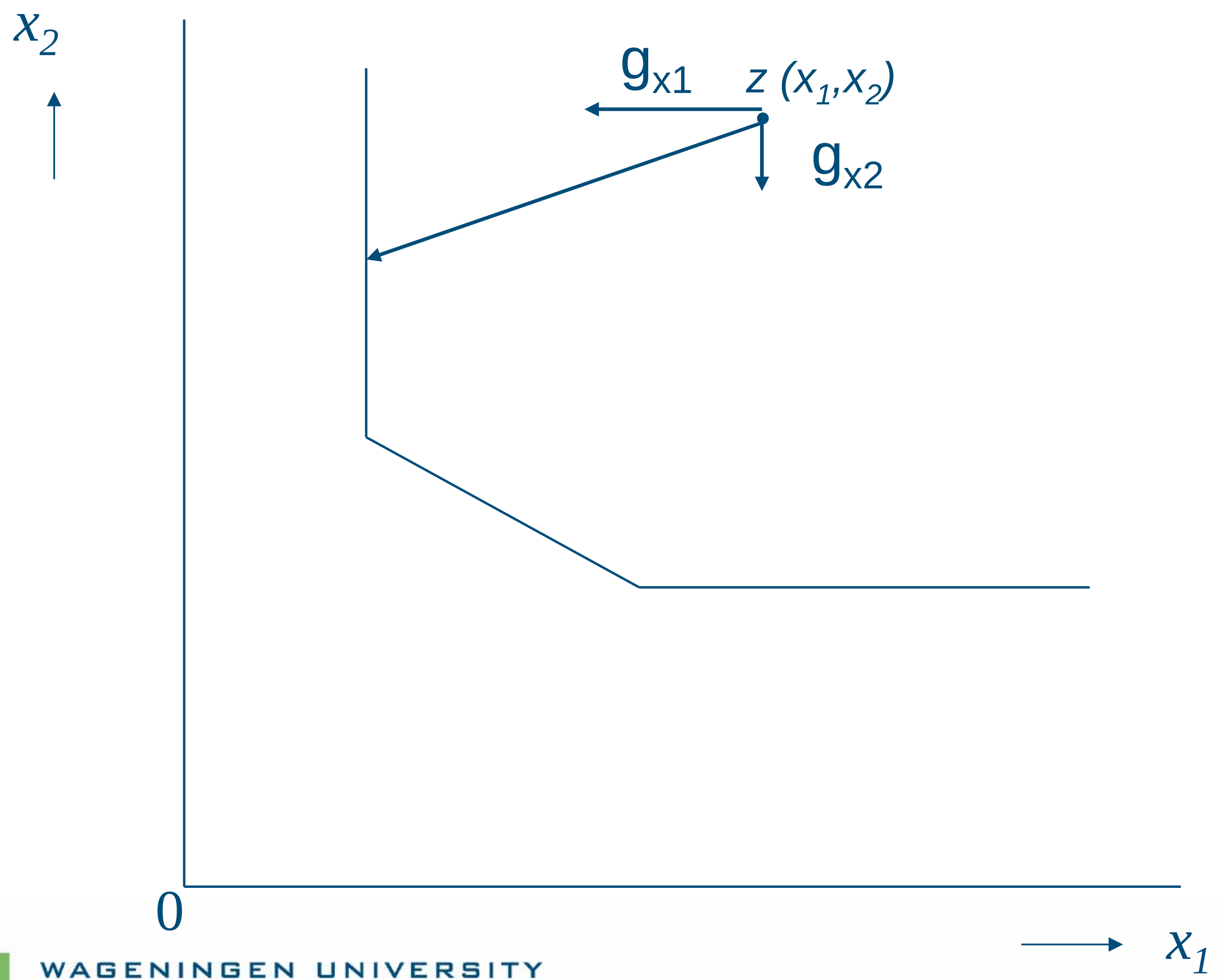
- The latter generates dynamic efficiency measures based on the production technology. The duality between this function and the optimal value function is developed and allocative efficiency measures are subsequently derived.

- Our work contributes to previous literature by parametrically estimating the dynamic directional distance function and the dual dynamic cost function.
- Our study measures:
  - Dynamic technical efficiency
  - Dynamic allocative efficiency

Radial Distance Function and Technical Efficiency



Directional Distance Function and Technical Efficiency



Dynamic Directional Distance Function

- Let **y**, **x**, **K**, **I** and **L** be a vector of outputs, variable inputs, capital stock, gross investments and fixed inputs, respectively. The input-oriented dynamic directional distance function can be defined as follows:

$$\vec{D}^i(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{x},\mathbf{I};\mathbf{g_x},\mathbf{g_I}) = \max\left\{\beta \in \Re : \left(\mathbf{x} - \beta\mathbf{g_x}, \mathbf{I} + \beta\mathbf{g_I}\right) \in V(\mathbf{y} : \mathbf{K},\mathbf{L})\right\},$$

$$\mathbf{g_x} \in \Re^N_{++},\ \mathbf{g_I} \in \Re^F_{++}, \left(\mathbf{g_x},\mathbf{g_I}\right) \neq \left(\mathbf{0}^N, \mathbf{0}^F\right)$$

- The distance function is a measure of the maximal translation of (**x**,**I**) in the direction defined by ($\mathbf{g_x}$,$\mathbf{g_I}$), that keeps the translated input combination inside the input requirement set.

Dynamic directional distance function

- It is assumed that firms are intertemporally cost minimizing:

$$W(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{w},\mathbf{c}) = \min_{\mathbf{x},\mathbf{I}} \int_t^{\infty} e^{-rt}\left[\mathbf{w}'\mathbf{x} + \mathbf{c}'\mathbf{K}\right]dt$$

$$s.t.$$

$$\dot{\mathbf{K}} = \mathbf{I} - \boldsymbol{\delta}\mathbf{K}$$

$$\vec{D}^{i}(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{x},\mathbf{I};\mathbf{g}_{\mathbf{x}},\mathbf{g}_{\mathbf{I}}) \geq 0$$

- where **w**, **c**, **δ** and *r* are variable input prices, capital rental prices, depreciation rates and the discount rate, respectively.

Dynamic directional distance function

- The dynamic cost inefficiency can be expressed as:

$$OI^i = \frac{\mathbf{w}'\mathbf{x} + \mathbf{c}'\mathbf{K} + W_{\mathbf{k}}(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{w},\mathbf{c})'(\mathbf{I} - \boldsymbol{\delta}\mathbf{K}) - rW(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{w},\mathbf{c})}{\mathbf{w}'\mathbf{g}_{\mathbf{x}} - W_{\mathbf{k}}(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{w},\mathbf{c})\mathbf{g}_{\mathbf{I}}} \geq D^i(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{x},\mathbf{I};\mathbf{g}_{\mathbf{x}},\mathbf{g}_{\mathbf{I}})$$

- *OI* , the cost inefficiency, is the difference between the observed shadow cost of input use and the minimum shadow cost, normalized by the shadow value of the direction vector.
- *TI* , technical inefficiency of both variable and quasi-fixed inputs, is measured by *D*.
- *AI =OI-TI* , the allocative inefficiency, is the difference between dynamic cost inefficiency and dynamic technical inefficiency.

Estimation

- The dynamic directional input distance function can be estimated using stochastic estimation techniques:

$$0 = \vec{D}_h^i(y, \mathbf{K}, \mathbf{L}, \mathbf{x}, \mathbf{I}, t; \mathbf{1}, \mathbf{1}) + \varepsilon_h \quad \varepsilon_h = v_h - u_h$$

- In order to estimate this expression, the translation property is used:

$$-\alpha_h = \vec{D}_h^i(y, \mathbf{K}, \mathbf{L}, \mathbf{x} - \alpha_h, \mathbf{I} + \alpha_h, t; \mathbf{1}, \mathbf{1}) + \varepsilon_h$$

- Stochastic estimation is accomplished by maximum likelihood procedures

$$\sigma_\varepsilon = \left(\sigma_u^2 + \sigma_v^2\right)^{1/2} \quad \lambda_\varepsilon = \sigma_u / \sigma_v$$

Estimation

- Point estimates of each producer’s technical inefficiency can be derived as follows:

$$TI_h^i = 1 - \frac{\mathbf{X}_h'\mathbf{A} - u_h}{\mathbf{X}_h'\mathbf{A}}$$

- where **x** and **A** are the vectors of explanatory variables and parameter estimates respectively, and *u* is replaced by its conditional expectation.
- One can obtain the dynamic cost inefficiency model by estimating the dynamic cost frontier model:

$$C_h = rW(y,\mathbf{K},\mathbf{L},w_2,\mathbf{c},t) - W_{\mathbf{k}}(y,\mathbf{K},\mathbf{L},w_2,\mathbf{c},t)\dot{\mathbf{K}} - W_t(y,\mathbf{K},\mathbf{L},w_2,\mathbf{c},t) + \xi_h$$

$$\xi_h = \gamma_h + \delta_h$$

Estimation

where $C_h = \dfrac{\mathbf{w'x} + \mathbf{c'K}}{w_1}$ is the observed long-run cost normalized by the variable input price $w_{1.}$

- Point estimates of each producer's overall inefficiency can be generated as follows:

$$OI_h = \left[1 - \frac{\mathbf{X}_h'\mathbf{B}}{\mathbf{X}_h'\mathbf{B} + \delta_h}\right] \Big/ \left[w_2 - W_{\mathbf{k}}(y, \mathbf{K}, \mathbf{L}, \frac{w_2}{w_1}, \frac{\mathbf{c}}{w_1}, t)\right]$$

- where **X** and **B** are the vectors of explanatory variables and parameter estimates, and δ is replaced by its conditional expectation.

Empirical application

- Specialized Dutch dairy farms : milk sales represent at least 80% of total farm income (2,614 observations on 639 farms).

- One output (total revenues), two variable inputs (variable costs other than feed and feed), two quasi-fixed inputs (breeding livestock and machinery and buildings) and two fixed inputs (land and labor).

- A quadratic specification is chosen for the distance and cost functions.

Results: directional distance function

Variable	Mean	Standard deviation
dD/dy	-7.41E-01	2.46E-02
dD/dI_1	-1.20E-01	1.19E-02
dD/dI_2	-1.24E-02	6.59E-03
dD/dx_1	4.75E-01	1.61E-02
dD/dx_2	3.92E-01	1.71E-02
dD/dK_1	9.20E-02	2.94E-02
dD/dK_2	6.50E-03	1.11E-02
dD/dL_1	5.20E-02	2.07E-02
dD/dL_2	1.34E-03	1.44E-02

- 70% of the parameters of the directional distance function is significant
- Dynamic technical inefficiency
  - decreases with output and investments
  - Increases with all inputs

Results: Cost Frontier

Variable	Mean	Standard deviation
dC/dK_1	-1.43E-03	2.40E-02
dC/dK_2	-4.19E-03	2.24E-02
dC/dw_2	4.37E-01	1.25E-01
dC/dc_1	1.61E-01	5.49E-02
dC/dc_2	1.16E-01	4.09E-02

- More than 50% of the parameters of the cost frontier is significant
- Long run costs increase with variable and quasi fixed factor input prices, and decrease with the size of the capital stock.

Results: Inefficiencies

Variable	Mean	Standard deviation
TI	0.107	0.105
OI	0.117	0.094
AI	0.010	0.097

- Technical inefficiency less than 11%
- Allocative inefficiency 1%
- 12% cost savings can be obtained

Concluding remarks

- Our analysis contributes to the literature by parametrically estimating dynamic efficiency measures based on a directional distance function and the duality between this function and the optimal value function.

- We propose an econometric estimation of dynamic overall, technical and allocative inefficiency measures.

- The empirical applicability of this proposal is illustrated by assessing dynamic efficiencies for a sample of Dutch dairy farms observed over the period 1995-2005.

Concluding remarks

- Dynamic efficiency ratings are compatible with static ratings derived by previous research.
- Average dynamic cost inefficiency indicates the possibility to accomplish long-run cost savings on the order of 12%.
- These cost savings are to be mainly achieved through a reduction in input use. An improved input mix given market prices offers, on the contrary, little scope for cost reduction.