Returns to Scale and Structural Change

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Motivation

- Suppose returns to scale evolves over time, increasing
- What are the implications for economic growth ?
- What are the implications for structural change?

Two past streams of literature

RTS and economic growth

- Solow (1956), Diamond (1965)

  DRTS ➔ convergence steady state

  CRS ➔ steady state only at origin, all trajectories conv to balanced growth paths
- Solow (1997) ~ CRS unlikely
- Endog growth lit

  IRTS ➔ stable, interior steady states with unbounded growth or decline leading to poverty traps

Two past streams of literature

RTS & Industrial Structure

- Baumol (1983, 1988) IRTS and structure when markets are contestable
- Winter et al. (2006) Heterogeneous firms, continuous stochastic entry
- Loyland and Ringstad (2001) structural effects of scale-augmenting tech change for Norwegian dairy industry estimate # farms would be reduced by 85% in absence of policy

Key Results from Lit

Questions and this paper

Questions

- How does RTS affect the structure of a network of interdependent enterpr ises?
- How does network structure affect growth?

This paper

- Dynamic simulation approach to structural change
  - Resource use change over time
  - Consideration of interdependence across multiple enterprises
  - Consideration of environmental processes, e.g. pollution
  - Illustration of the role of RTS in above

- Approach
  - Consider of small network of interdependent enterprises
  - Numerical simulation under varying RTS cases

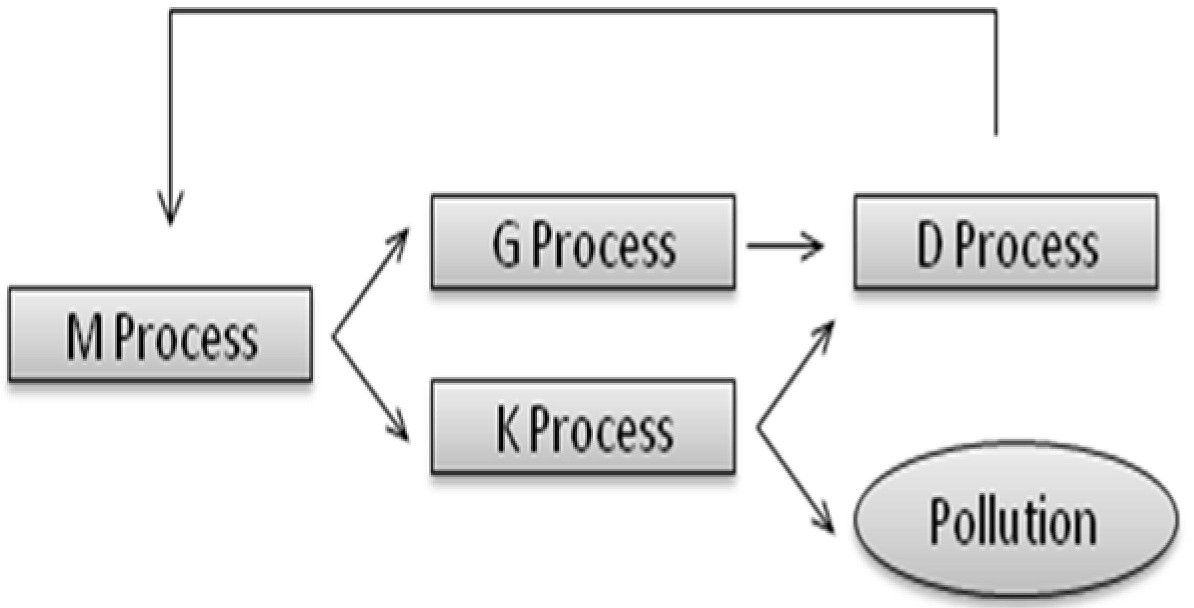
Interdependence

- Interdependence
  - Intermediacy of goods produced (vertical)
  - Externality production (horizontal, vertical, spatial)
  - Joint dependence on common resources
  - Differential interest in network member and network performance.
- Complexity
  - Discrete and continuous time dynamic processes
- Dynamics
  - Process dynamics
  - Investment
  - Adjustment costs

Approach

- Sylized multiple enterprise growth model
- Interpretation
  - Multi-shop supply network feeding OEM
  - Dairy farm with integrated dairy, field crop, and pasture enterprises
- Numerical computational solutions & simulation

Small network process architecture



Field of operations

- Suppose there are multiple “shops” (a.k.a. fields)
- Suppose each shop specializes in one of a vector of outputs (crops)
- Shops supply an OEM (dairy)
- OEM produces intermediate good for shops (manure) that is a potential source of pollution
- Shops purchase inputs and use OEM’s intermediate good to augment productivity (manure + fertilizer➔ nitrogen)
- But….use of those inputs yields pollution

Discrete time process (t clock) - a.k.a. crops

K Process *j th shop (field) i th product (crop)*

$$y_{k,i}^{j}(t) = A_{k,i}^{j}(t)\left(Z_{k,i}^{j}(t)\right)^{\alpha_1}\left(x_{m,i}^{j}(t)\right)^{\alpha_2}\theta_{k,i}^{j}(t)$$

$$\dot{A}_{k,i}^{j}(t) = a_0 x_{m,i}^{j}(t) A_{k,i}^{j}(t) - a_y y_{k,i}^{j}(t)$$

tech & controlled augmentation use based diminution

$$\dot{I}_{k,i}^{j}(t) = y_{k,i}^{j}(t) - s_{k,i}^{j}(t) - x_{k,i}^{j}(t)$$

Characterizing RTS and dynamics

$$\phi(\lambda, t) = \lambda^{\alpha_1(t)+\alpha_2(t)}$$

$$\psi(t) = (\partial\phi(\lambda,t)/\partial\lambda)(\lambda/\phi(\lambda,t)) = \alpha_1(t) + \alpha_2(t)$$

$$\dot{\psi}(t) = v_1\dot{\alpha}_1(t) + v_2\dot{\alpha}_2(t) \quad where \quad \dot{\alpha}_1(t) = \dot{\alpha}_2(t) \quad and \quad v_q = \alpha_q/(\alpha_1 + \alpha_2)$$

Thus, process output dynamics can be expressed in terms of RTS dynamics.

Continuous process enterprises (*d=1,….D)*

D Process (tau clock)

$$y_d(\tau) \equiv L_{d,}(\tau) Z_d^{\xi_1}(\tau) x_f^{\xi_2}(\tau)$$

$$\xi_1 + \xi_2 \leq 1$$

Vintage Function

$$L_d(\tau) \equiv \mu v_d^{\omega_1}(\tau) e^{\omega_2 v_d}$$

$$v_d(\tau) = \tau - \tau_d$$

Joint output as a source of pollution

D Process

$$y_m(\tau) \equiv H * Z_m^{\beta_1}(\tau) * x_f^{\beta_2}(\tau),\ \beta_1 + \beta_2 \leq 1$$

$$\dot{I}_m(\tau) = y_m(\tau) - s_m(\tau) - x_m(\tau)$$

$x_f(\tau)$ is an input to M and D process which is written as

$$x_f(\tau) \equiv \lambda_k \sum_i x_{k,i}(t) + \lambda_g x_g(\tau) + \lambda_c x_c(\tau)$$

Pollution

$$x_m(t) \equiv \sum_i \sum_j x^j_{m,i}(t)$$

$$x_m(t) \leq \sum_m I_m(t)$$

- Potential pollution via intermediate flow from D process

$$y_{n,i}(\tau) \equiv n_f\left(x_m(t)+1\right)\left(x_f(\tau)+1\right)+n_k\left(\sum_j Z^j_{k,i}(t)\right)$$

Applications : Pollution Control

- Pollution as residual of potential – recycling (uptake)

$$e_{n,i}(\tau) \equiv y_{n,i}(\tau) - u_{k,i}(\tau), \quad i = 1, 2, ..., n$$

where $u_{k,i}(\tau)$ is the usage of by-products, which can be written as

$$u_{k,i}(\tau) = u_{kk} * v_k(\tau) * \sum_j y_{k,i}^j(t)$$

$$v_k(\tau) = 1/\tau^2$$

Continuous process by shops (grass)

$$y_g(\tau) = A_g(\tau) * (Z_g(\tau))^{\gamma_z} * (x_m(\tau))^{\gamma_x}$$

$$\dot{A}_g(\tau) = a_{g,0} x_m(\tau) A_g(\tau) - a_{g,y} y_g(\tau)$$

$$\dot{I}_g(\tau) = y_g(\tau) - s_g(\tau) - y_c(\tau)$$

Intermediate use based on processing (cutting)

$$y_c(\tau) \equiv \dot{I}_c(\tau) + s_c(\tau)$$

Network profits

$$\pi(\tau) \equiv \sum_t \sum_i P_k s_{k,i}(t) + \int_{\tau_0}^{\tau_f} \sum_m P_m s_m(\tau) d\tau + \int_{\tau_0}^{\tau_f} \sum_d P_d s_d(\tau) d\tau$$

Revenue

$$+ \int_{\tau_0}^{\tau_f} \sum_c P_c s_c(\tau) d\tau - \sum_t \sum_k R_k Z_k(t) - \int_{\tau_0}^{\tau_f} \sum_m R_m Z_m(\tau) d\tau$$

Input cost

$$- \int_{\tau_0}^{\tau_f} \sum_d R_d Z_d(\tau) d\tau - \int_{\tau_0}^{\tau_f} \sum_k c_k I_k^{c_1}(\tau) d\tau - \int_{\tau_0}^{\tau_f} \sum_m c_m I_m^{c_2}(\tau) d\tau$$

Inventory cost

$$- \int_{\tau_0}^{\tau_f} \sum_n c_n I_n(\tau) d\tau - \int_{\tau_0}^{\tau_f} \sum_g c_c I_c(\tau) d\tau - \int_{\tau_0}^{\tau_f} \sum_i k\left(b_{n,i}(\tau), e_n(\tau)\right)$$

Environmental cost

$$- \int_{\tau_0}^{\tau_f} \sum_i c_u u_{k,i}(\tau) - \int_{\tau_0}^{\tau_f} \sum_n P_n e_n(\tau) d\tau$$

Control problem

$$w(\tau) \equiv w_1 \sum_{\tau} e_n(\tau) + w_2 \sum_{\tau} \left(e_n(\tau)\right)^2 + w_3 \sum_{\tau} \pi(\tau)$$

$$J = \max \int_{\tau_0}^{\tau_f} w(\tau) d\tau$$

Subject to dynamics of processes

Summary of notation

Table : Inputs and outputs of each process.

Process	Input	Output	Sell	Intermediate	Inventory
K	x_m, Z_k^j	y_k^j	s_k^j	x_k	I_k
M	x_f, Z_m	y_m	s_m	x_m	I_m
D	x_f, Z_d	y_d	N/A	N/A	N/A
N	x_f, x_m, Z_k^j	y_n	e_n^j (by-products)	N/A	I_n
G	x_m, Z_g	y_g, y_c	s_c	x_g, x_c	I_g, I_c

Parameterization

- Prices, unit costs

Parameters	P_k	P_m	P_d	P_c	P_n	w_1	w_2	w_3	c_1
Value	15	8	30	10	0/Varies[1]	0/-0.9[1]	0/-0.2[1]	1	0.7
Parameters	R_k	R_m	R_d	R_g	c_k	c_m	c_n	c_c	c_2
Value	10	3	5	7	0.5	2	0.9	2	0.19

Table 2: Parameters used in simulation

Parameterization

Process				
K	Batch (crops)	Alpha1=0.5	Alpha2=0.3	
Adot	“	a0 = 0.0001	a4=0.0004	
D	Continuous (milk)	eta1=0.5	etak1=0.29 etak2=0.07 etak3=0.01	
Ld	Vintage (lactation)	Mu=0.5	W1=0.68	W2=0.6
M	Potential poll (manure)	Beta1=0.5	Beta2=0.29	
Xf	Intermed (feed)	Lambdak=0.5	Lam g=0.25	Lam c=0.25
Yn	Pollutant	Nf=0.01	Nk=0.1	Cn=0.9
Yg	Continuous (grass)	Gamma z=0.5	Gamma x=0.3 .	
Ag dot	“	Ag0=0.00005	Agy=0.0003	

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Cases Illustrated

CASE 1: Near Constant returns to scale, $\psi(t) = 0.99$.
CASE 2: Decreasing returns to scale $\psi(t) = 0.80$, no abatement.
CASE 3: Decreasing returns to scale $\psi(t) = 0.50$, no abatement.

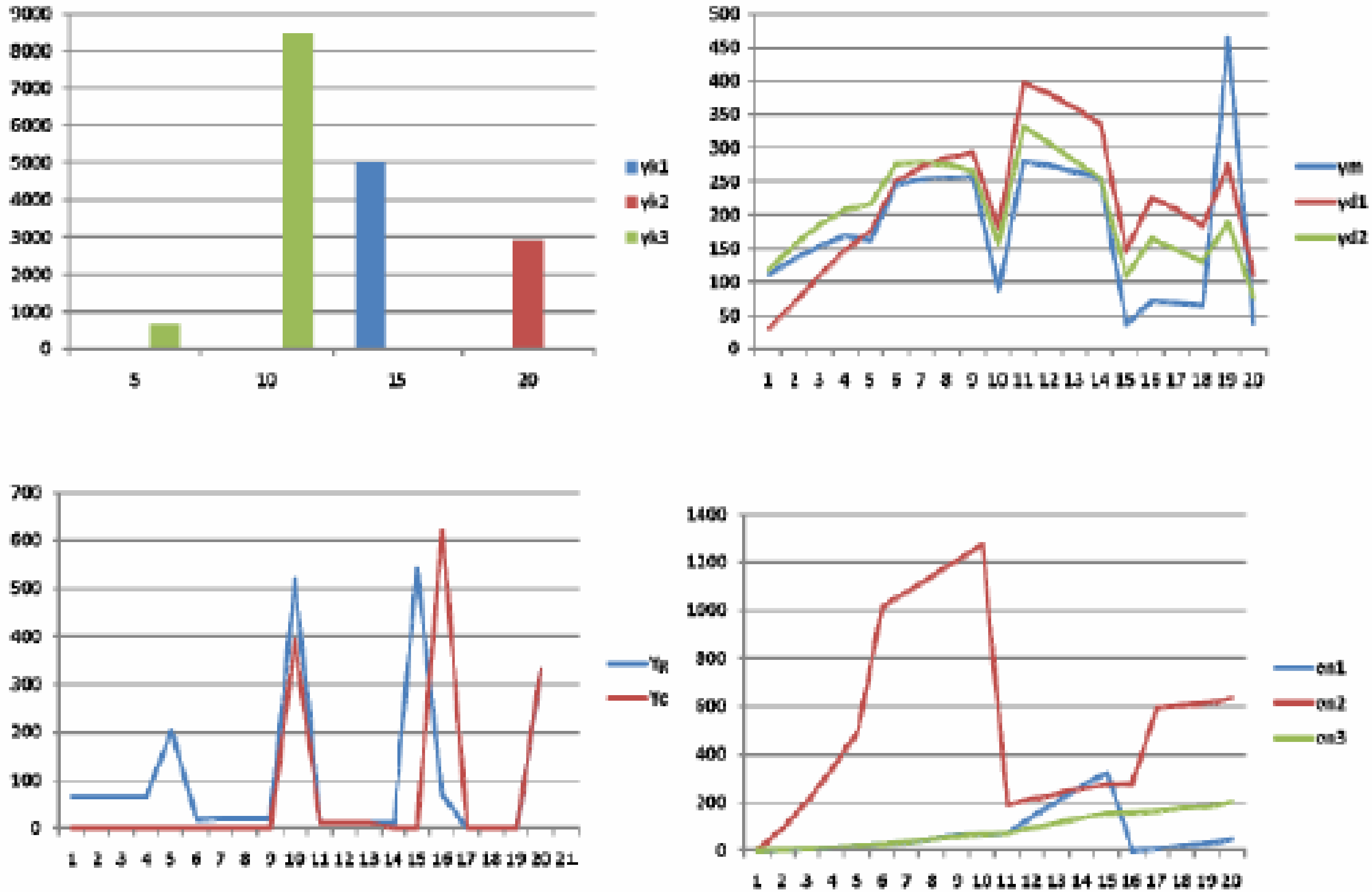


FIGURE 2. Case 1a. Output and Pollution Dynamics

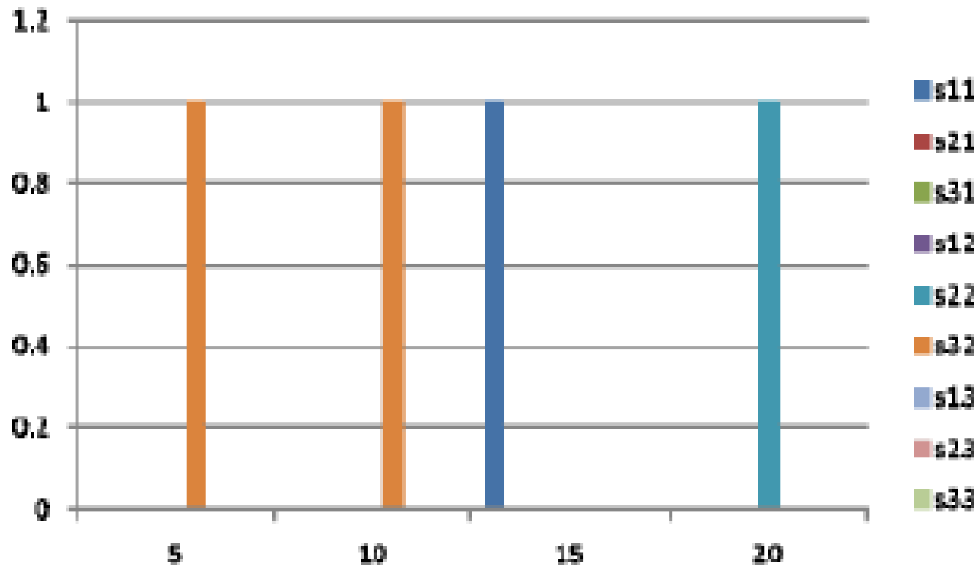
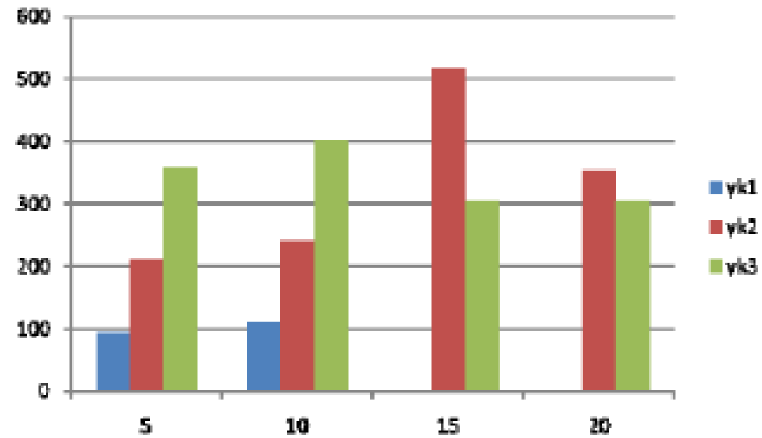
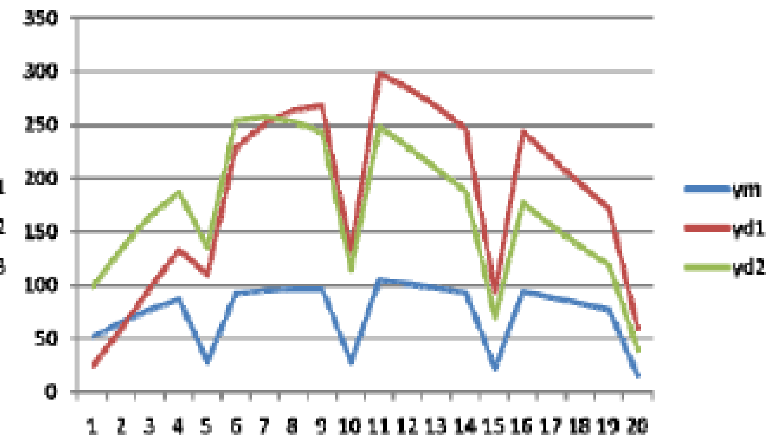
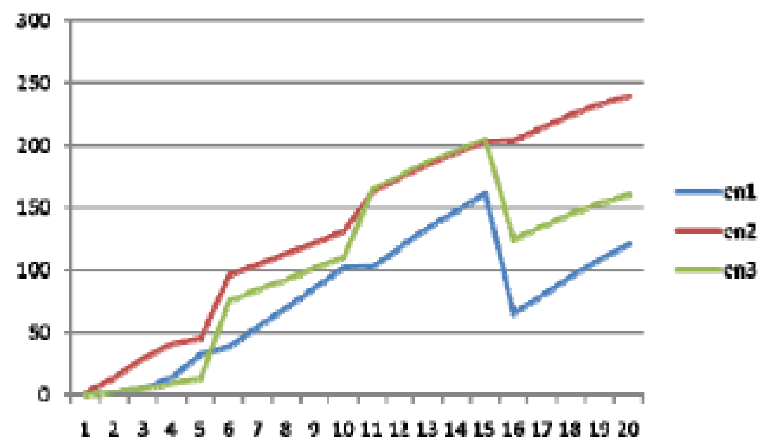


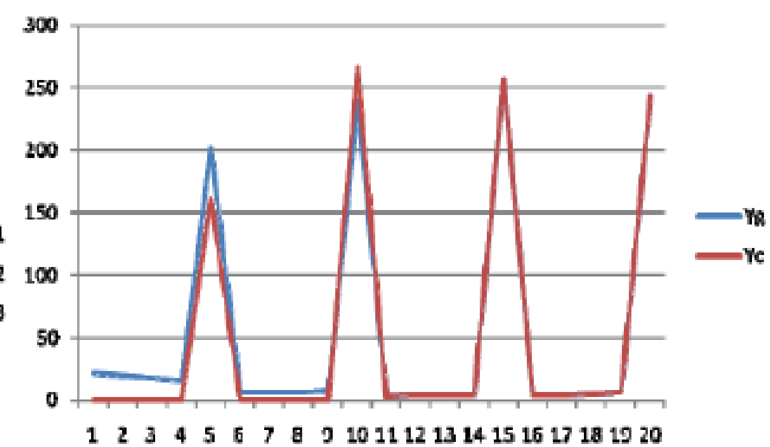
FIGURE 3. Case 1a Optimal scales of operation
$\theta^{j}_{k,i}(t)$ = sij = scale of i^{th} product by j^{th} shop.





DRTS ➔ diversification, intensification





Cont prod for sale, not intermediate use

Pollution becomes unstable

FIGURE 4 CASE 2: Decreasing returns to scale $\psi(t) = 0.80$.

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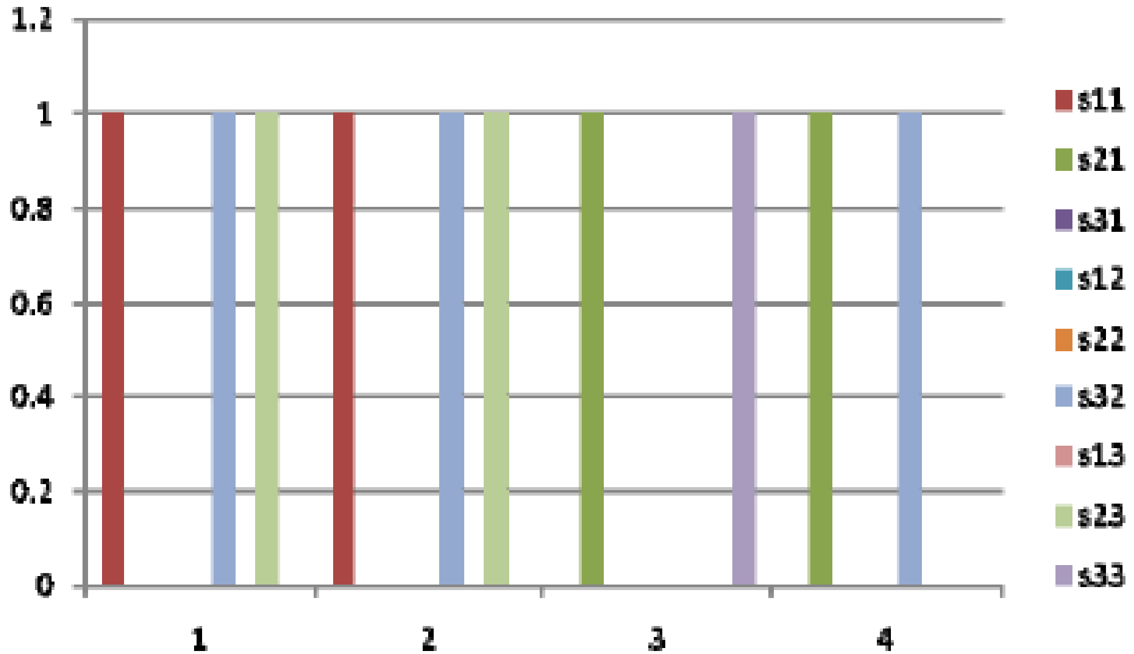


FIGURE 5 CASE 2: Decreasing returns to scale $\psi(t) = 0.80$.
Optimal scales of operation $\theta^{j}_{k,i}(t)$ = sij = scale of i^{th} product by j^{th} shop.

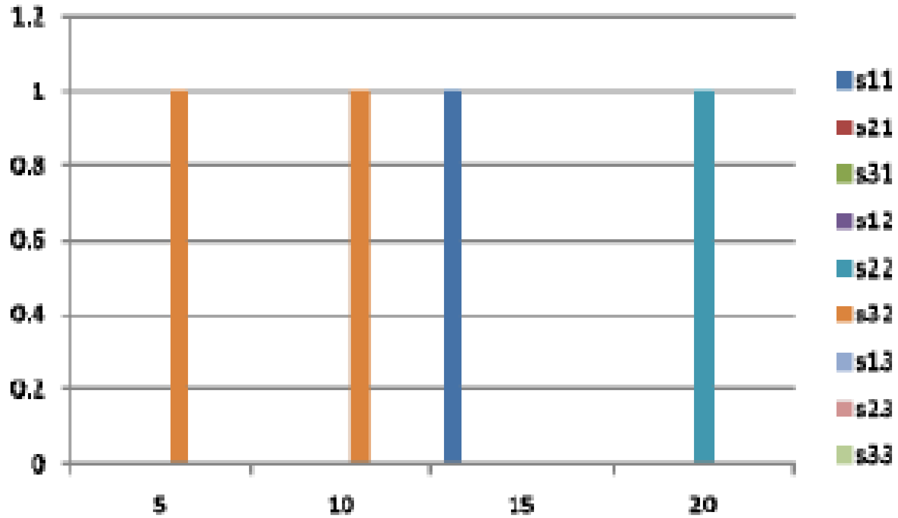
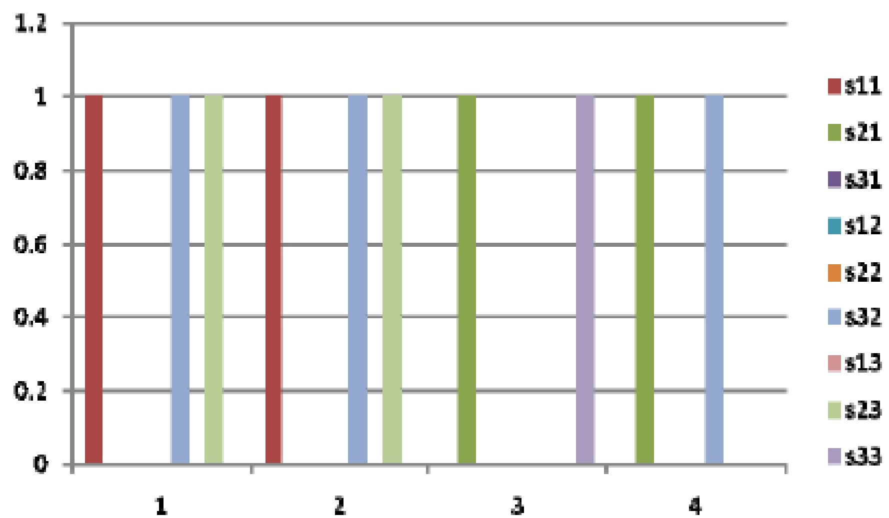


FIGURE 3. Case 1a Optimal scales of operation

$\theta^{j}_{k,i}(t)$ = sij = scale of



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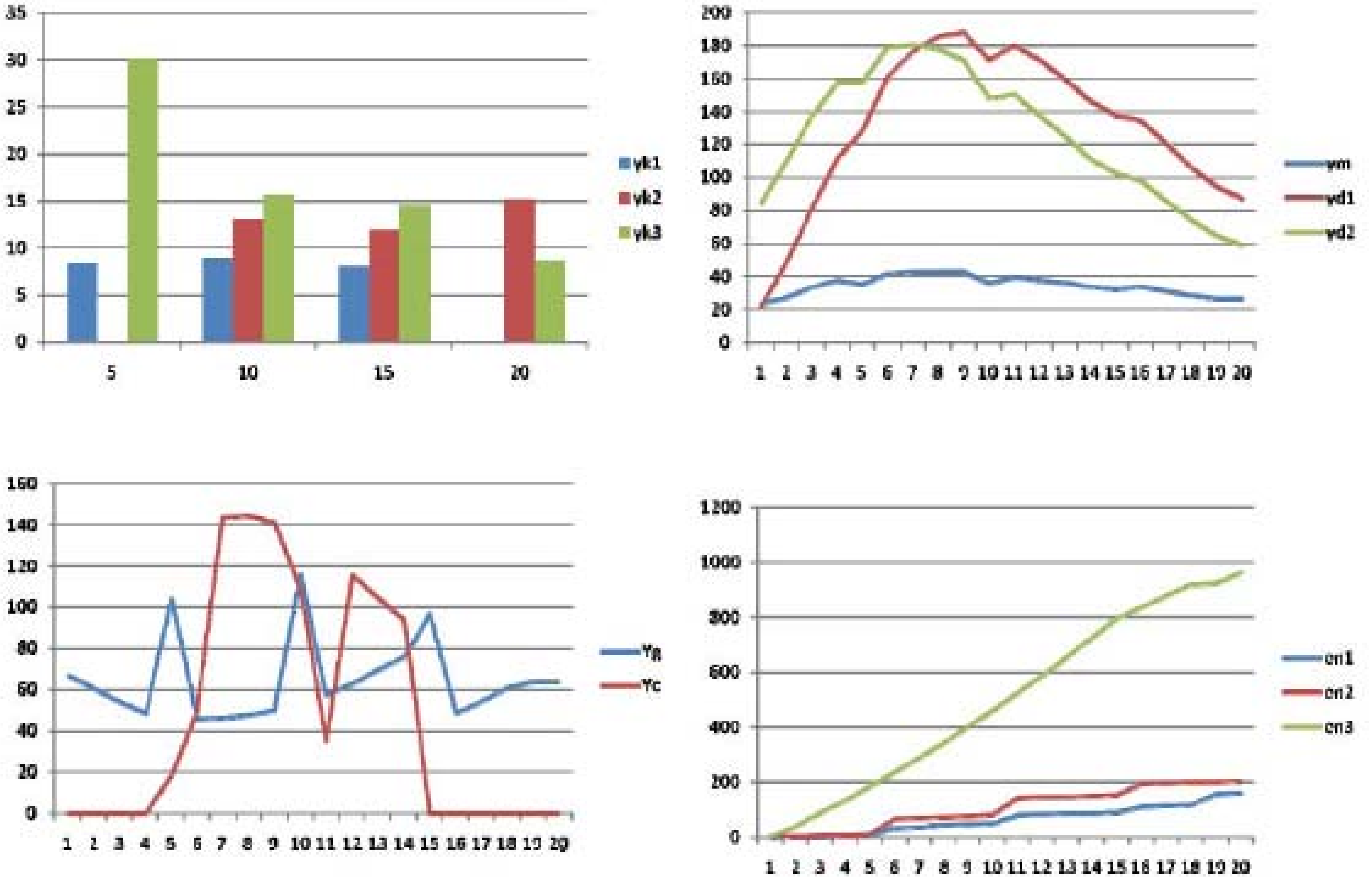

FIGURE 6 CASE 2 Decreasing returns to scale $\psi(t) = 0.50$.

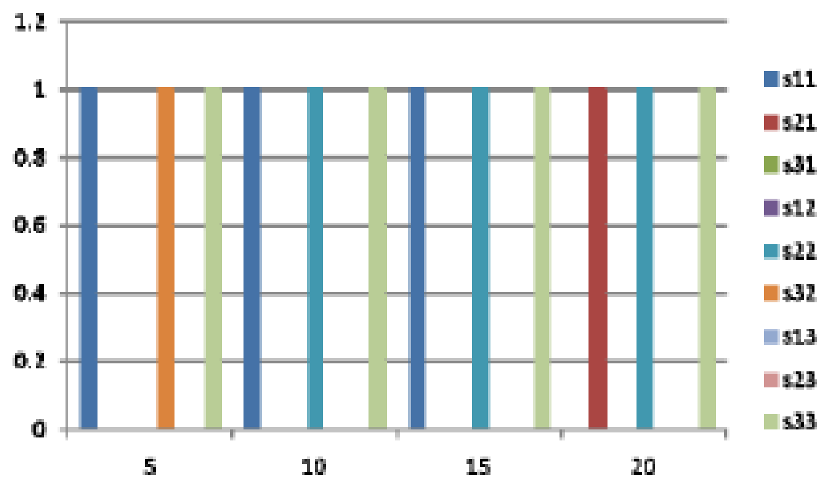

FIGURE 7 CASE 2 Decreasing returns to scale $\psi(t) = 0.50$.
Optimal scales of operation $\theta^{j}_{k,i}(t)$ = sij = scale of i^{th} product by j^{th} shop.

Structural Implications

- Resource use transition and dynamics is analyzed
- As RTS increases specialization increases