A Dynamic Model under State-contingent Production Uncertainty

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Introduction

- The influence of risk on agricultural production decisions has been addressed widely in the literature. Pope and Chavas (1994) demonstrate that, under risk aversion, cost minimization cannot be adequately characterized by expected output alone.
- Chambers and Quiggin (1998, 2000) represent the stochastic technology using a *state-contingent* input correspondence.
- They show that under a state-contingent approach a standard cost minimization problem applies irrespective of risk preferences.
- The state-contingent approach is based on the assumption that production under uncertainty can be represented by differentiating outputs according to the state of nature in which they are realized.

Introduction

- The state-contingent approach has seen very few empirical applications.
- Chavas (2008) is one exception: he develops a methodology to estimate cost-minimizing input choices.
- By working with a static cost minimization framework, Chavas (2008) assumes capital is a fixed input.
- The role of uncertainty on production decision making and investment patterns remains an open question.
- The use of a state-contingent framework is particularly useful to introduce production risk in dynamic models, since their complexity makes it difficult to model risk and risk attitudes by means of an expected utility model.

Introduction

- In this paper we advance a dynamic state-contingent cost minimization approach.
- We assess production decisions in US agriculture over the last century and determine how the costs of producing under different states of nature have changed over time.
- We also provide insights on the role of risk on US agriculture capital accumulation patterns, which has not been studied using the state-contingent methodology.

The model

- Uncertainty is represented by a series of states of nature $\Omega = \{1,\ldots,S\}$.
- Let $y = \{y_1,\ldots,y_s\}$ be the output realized under Ω .
- The intertemporal cost minimization problem is:

$$V(\mathbf{w},\mathbf{c},\mathbf{k},\mathbf{y}) = \min_{x,I} \int_t^{\infty} e^{-rt}\left[\mathbf{w}'\mathbf{x}(t) + \mathbf{c}'\mathbf{k}(t)\right]dt$$

$$s.t.$$

$$\dot{\mathbf{k}} = \mathbf{I} - \delta\mathbf{k}$$

$$\mathbf{y}(t) = F\left[\mathbf{x}(t),\mathbf{k}(t),\mathbf{I}(t)\right] \quad t \in \left[t,\infty\right)$$

where:
V = long-run cost function,
x (w) = variable input quantities (prices),
K (c) = quasi-fixed input quantities (prices),
I = gross investments ,
δ= diagonal matrix containing depreciation rates,
r = interest rate and *F* = transformation function.

The model

- The Hamilton-Jacobi-Bellman equation is:

$$rV(\mathbf{w},\mathbf{c},\mathbf{k},\mathbf{y}) = \min_{x,I}\left[\mathbf{w}'\mathbf{x} + \mathbf{c}'\mathbf{k} + (\mathbf{I} - \delta\mathbf{k})' V_k(\mathbf{w},\mathbf{c},\mathbf{k},\mathbf{y})\right] + \varphi\left[\mathbf{y} - F(\mathbf{x},\mathbf{k},\mathbf{I})\right]$$

- The first derivatives of HJB with respect to input prices yield input demand equations:

$$\mathbf{x} = rV_w - \dot{\mathbf{k}}V_{kw}$$

$$\dot{\mathbf{k}} = V_{kc}^{-1}(rV_c - \mathbf{k}).$$

- Following Chambers and Quiggin, if actual input choices do not minimize cost, choosing input use according to these equations will improve the welfare of the decision maker irrespective of risk preferences:
- The standard cost minimization model is applicable independently of risk attitudes.

The model

- The empirical challenge is to measure the state-contingent output when only ex-post data are available. Chavas (2008) recovers the ex-ante technology by defining a new random variable, *e*, a deterministic transformation of *y* capturing the relative changes of output across states of nature:

$$e_s \equiv (y_s / \mu_t)^{1/\sigma_t}$$

- μ captures production technology and σ allows the spread of output distribution to vary across observations.
- Production uncertainty is measured by assuming an auxiliary variable that under state *s* satisfies the following condition:

$$\ln(z_t) = \ln(o_t) + \sigma_t \ln(e_t)$$

The model

- If at time *t* state *s* occurs, one can estimate the vector of *T* realized values of the random variable. The vector of state-contingent outputs can then be derived as:

$$\mathbf{y}_t^e = \left\{ y_{rt} : y_{rt} = y_t \frac{(z_r / o_r)^{\sigma_t / \sigma_r}}{(z_t / o_t)} ; r = 1, \ldots, T \right\}$$

Empirical specification

- Following Chavas, and to avoid multicollinearity, only two states of nature are distinguished.
- The estimation of the regression $\ln(z_t) = \ln(o_t) + \sigma_t \ln(e_t)$ employs a GARCH (1,1) specification.
- The dependent variable is the log of output on a per unit of land.
- The structural part of the model is specified as a function of an aggregate machinery and land price index and a fertilizer price index. A research and development expenditures index (RD) on a per unit of land, and the lagged dependent variable are also included.

Empirical specification

- We consider:

  $\mathbf{x} \in R_+^1$ (numeraire)

  $K \in R_+^2$

- Under this specification, the value function *V* depends on **y**, **c**, and **K,** where **c** is now a vector of normalized capital rental rates.
- Following Epstein (1981), *V* is specified as:

$$V(\mathbf{y}^L, \mathbf{c}, \mathbf{k}) = a_0 + (a_1' \quad a_2' \quad a_3') \begin{pmatrix} \log \mathbf{y}^L \\ \log \mathbf{c} \\ \mathbf{k} \end{pmatrix} + \left((\log \mathbf{y}^L)' \quad (\log \mathbf{c})' \quad (\mathbf{k})' \right) \begin{pmatrix} \mathbf{A} & \mathbf{F} & \mathbf{G} \\ \mathbf{F}' & \mathbf{N} & \mathbf{0} \\ \mathbf{G}' & \mathbf{0} & \mathbf{D} \end{pmatrix} \begin{pmatrix} \log \mathbf{y}^L \\ \log \mathbf{c} \\ \mathbf{k} \end{pmatrix} + \mathbf{c}' \mathbf{M}^{-1} \mathbf{k}$$

Empirical specification

- The conditional demands for the variable and quasi-fixed inputs are:

$$x_n^* = ra_0 + ra_1' \log \mathbf{y}^L + ra_2'(\log \mathbf{c} - (\hat{\mathbf{c}}^{-1})' \mathbf{c}) + r\left(\frac{1}{2}\log \mathbf{c}' - \mathbf{c}'\hat{\mathbf{c}}^{-1}\right)\mathbf{N} \log \mathbf{c} +$$

$$r\left(\log \mathbf{c}' - \mathbf{c}'\hat{\mathbf{c}}^{-1}\right)\mathbf{F}' \log \mathbf{y}^L + a_3'(r\mathbf{k} - \dot{\mathbf{k}}) + \left(\frac{r}{2}(\log \mathbf{y}^L)' \mathbf{A} \log \mathbf{y}^L\right) + \mathbf{k}'\mathbf{D}\left(\frac{1}{2}r\mathbf{k} - \dot{\mathbf{k}}\right) +$$

$$(\log \mathbf{y}^L)' \mathbf{G}(r\mathbf{k} - \dot{\mathbf{k}})$$

$$\dot{\mathbf{k}}^* = r\mathbf{M}\hat{\mathbf{c}}^{-1}(a_2 + \mathbf{F}' \log \mathbf{y}^L + \mathbf{N} \log \mathbf{c}) + (r\mathbf{U} - \mathbf{M})\mathbf{k}$$

Empirical application

- Our model is applied to US agriculture over the period 1910-1990. We ask how the costs of facing different production risks have been changing over time when accounting for the quasi-fixity of assets. We also provide insights on the impacts of these risks on investment decisions.
- An augmented version of the dataset found in Thirtle et al. (2002) is used to estimate the model.
- Input and output variables are defined as follows.

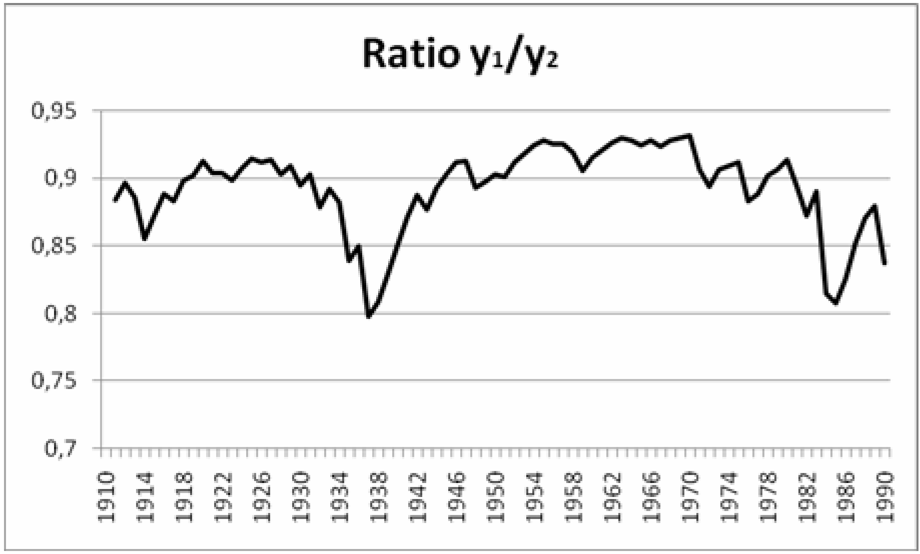
– k_1 = labor, k_2 = land and machinery, with prices c_1 and c_2

– x = fertilizer, with price w

– Y = aggregate agricultural production.

Results: state-contingent output simulation

- The ratio y_1/y_2 (unfavorable to favorable yields) shows a strong downward trend during:
  - the Great Depression and
  - the farm financial crisis of the early-to-mid 1980s.
- During difficult economic times, the output obtained under favorable states grows quicker than the output under less advantageous conditions, which may be the result of firms adopting more conservative production practices.

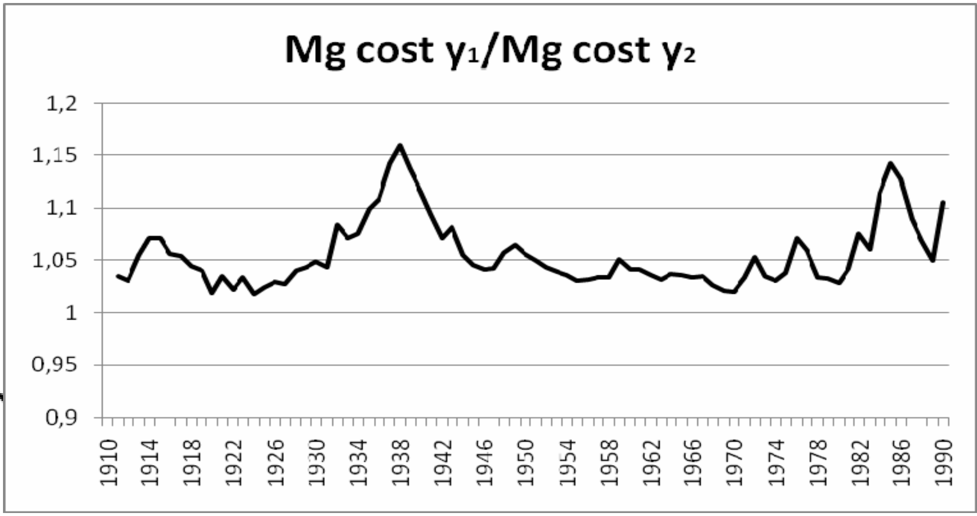


Results: marginal costs

- Input demand equations are estimated by SUR.
- Estimated capital adjustment rates are:

$$(r\mathbf{U} - \mathbf{M}) = \begin{pmatrix} -0.114 & \\ -0.022 & -0.022 \end{pmatrix}$$

- The negative semidefiniteness of the matrix guarantees convergence (though very slow) to the equilibrium.
- The ratio of marginal cost of y_1 and y_2 has a negative correlation with the output ratio.
- It also indicates that producing under unfavorable states is marginally more expensive than producing under more favorable ones



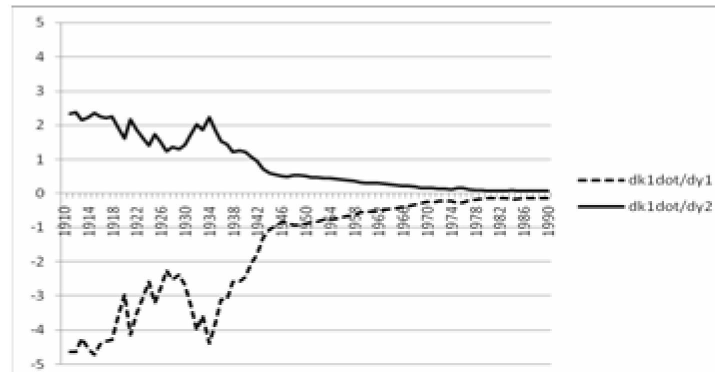
Results: marginal costs

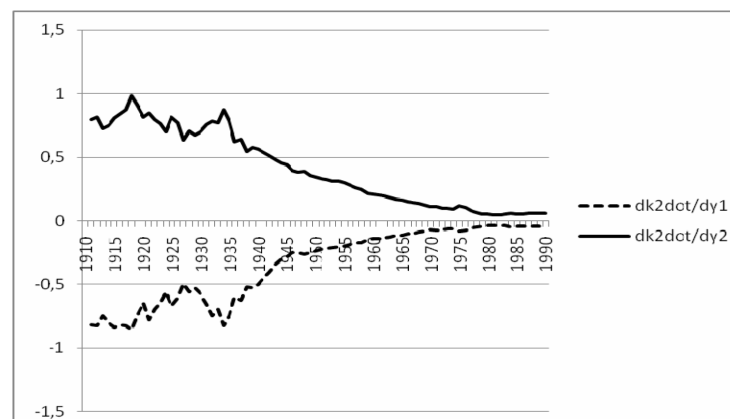
- The evolution of long-run marginal costs of y_1 and y_2 suggests improvements in technology that have reduced marginal costs.
- The dynamics of labor has been specially relevant in reducing the marginal cost under unfavorable states.
- Marginal costs registered important increases during the Great Depression. Minimum levels were registered at the beginning of the 1980s.

- There is a decline in the distance between the two marginal costs. This suggests that farmers have adopted improved techniques focused on reducing the marginal costs of production under unfavorable production conditions.

Results: risk and investments

- These figures show the evolution of the first derivatives of net capital investments with respect to production.
- While bad states of nature (y_1) discourage investments, favorable conditions (y_2) have a positive impact.
- Differences between the impacts of good and bad states are specially pronounced at the beginning of the 20th century and the Great Depression and tend to diminish as we approach the end of the century.





Results: risk and investments

- The effects of output risk on asset acquisitions in agriculture, have tended to decline over time.
- In spite of reduced differential impacts of production risk on investments by the end of the period analyzed, good (bad) states of nature continue to encourage (discourage) farm investments in labor, machinery and land.
- Although the effects of good and bad states on investment are rather symmetric, the negative influence that bad states have on labor net investments is not compensated by the positive effect of good states.
- On the other hand, the good state effects for land and machinery investments are more powerful than the disinvestment impacts of bad states. Hence,

Concluding remarks

- Where is technical change?
- We represent the stochastic nature of production using the state-contingent approach proposed by Chambers and Quiggin (1998, 2000) and empirically implemented by Chavas (2008). A dynamic state-contingent cost minimization approach is applied to assess production decisions in US agriculture over the last century.
- Results suggest a tendency to reduce the output produced under unfavorable conditions during difficult economic times. Parameter estimates of the dynamic dual model indicate the presence of capital adjustment

Concluding remarks

- We find that marginal costs have a declining trend that is only reversed during difficult economic situations (Great Depression and 1980s farm financial crisis) when producing under unfavorable states of nature becomes more expensive, and firms take more conservative production decisions.
- Finally, we also show the impacts of production risk on farm investment decisions. Our results suggest that while good states of nature tend to encourage investments in quasi-fixed assets, bad states of nature discourage them.
- Differential impacts of different states of nature on net investments, however, have tended to decline over time as risk management techniques have been improving and the extra cost of producing under bad states relative to good ones, has been declining

Questions?