Returns to Scale and Dynamics of Multiple Enterprises

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ABSTRACT

This paper presents a dynamic theory of structural change in which functional change driven by technological change and transactional change open opportunity for change in scale and scope of enterprises. Implementation of change in scale and scope of enterprises is constrained by initial state conditions including resource endowments, access to credit, and regulation. In the presence of such constraints, the paper motivates the existence of thresholds that introduce cusps in the optimal paths of control variables.

The evolution of enterprises is considered within the context of a multiple enterprise firm that encompasses discrete and continuous processes that are interdependent on intermediate goods as well as their production of environmental effects. Response to opportunity is governed as well by adjustment costs and incomplete markets. The specification is consistent with manufacturing, crop and animal agriculture, as well as service-oriented enterprises. Potential for structural change is considered within the context of changes in prices, regulation, and technological change. We discuss management options that maximize uncertain profit over the planning horizon subject to changing price regimes, regulations, and existing and new technology. We consider incentive as well as quota type policy to regulate environmental effects. While the model is one of optimal control, the presence of constraints opens the opportunity for timing of adjustment and constrains adjustment. We derive thresholds for change and show their principal determinants. Based on this specification, we derive an indicator of flexibility as well as real options style valuation of the benefits of postponement of adjustment.

The model's specification is sufficiently complex as to preempt analytical consideration. Instead, we specify a numerical example and present illustration of the implications of the model based on numerical computation. The example illustrates both unbundling of enterprises as conditions encourage reduction in scope as well as formation of relational links to recover what might be viewed as economies-of-scale by specialized enterprises spawned by unbundling. The example also illustrates conditions necessary to induce a shift from traditional grazing animal operations integrated with crops to intensive feed lots. Within this context, shift in scale, specialization, and scope of enterprises is illustrated.

To conclude, the utility of model is considered within general contexts to analyze the structural implications of change in economic conditions such as shifts in price regimes, change in process technology, change in IT technology, and changes in institutional rules.

INTRODUCTION

This paper presents a dynamic theory of structural change in which functional change in returns to scale impact scale of enterprises. Change in returns to scale resulting from technological change or change in transaction methods are viewed as possible origins of change in returns to scale. Production choices are constrained by initial state conditions including resource endowments and technological productivity.

A deep stream of literature has recognized the central role that returns to scale play in growth dynamics. Solow (1956) and Diamond (1965) specified decreasing returns to scale over the reproducible factors of production and, from all points except the origin, found a steady state to which all trajectories converge. With constant returns to scale this result melts away leaving the origin as the only steady state and all trajectories converging to balanced growth paths. While this result have been found attractive to some, Solow's (1997) critique that the result may be of little import given the unlikely occurrence of constant returns to scale in the real world. In the endogenous growth literature, increasing returns to scale result in stable, interior steady states. One class of these results in unbounded growth while another set results in decay to the origin, a process that has been labeled as the poverty trap, see Shell (1966, 1967) or Azariadis and Drazen (1990). One source of increasing returns to scale in reproducible factors noted in the literature has been the presence externalities. Lucas (1988) and Romer (1990) exploit this specification to motivate constant returns to scale in societal level technology despite decreasing returns to scale in private technologies and, thereby, motivate endogeneous growth that is balanced. In such a case, an indeterminancy results in a continuum of equilibria, see e.g. Harrison (2001), Harrison and Weder (2000), or Benhabib and Farmer (1996). As Harrison (2001) notes, the existence of multiple equilbria has important implications such as self-fulfilling expectations that generate aggregate fluctuations. She shows that even small external effects can lead to such indeterminancy. While this discussion is of great interest, it has a narrow scope of focus failing to consider structural implications of returns to scale.

A second stream of literature has considered the role of returns to scale evolution and the dynamics of industrial structure. Baumol (1983, 1988) noted in a theory of contestible markets how the extent of increasing returns to scale determines industrial structure. Evidence of this role is found throughout the economy, see Gustavasson (2002). Winter, et al. (2006) present a model of dynamics of industrial structure based on heterogeneous firms and continuous, stochastic entry. In a review of empirical evidence, they note high heterogeneity, continuous and dynamic disequilibrium, and rapid turnover of SMEs. Detailed studies are limited that consider evidence of the role of scale or returns to scale on the structure of industries. Loyland and Ringstd (2001) consider the structural effects of scale-augmenting technical change on the Norwegian dairy industry and estimate the number of farms would have been reduced by 85% through full exploitation of scale economies that expanded optimal scale from 1972-1996 in Norway. Huffman and Evenson (2001) found empirical evidence that expanding farm size in the US from1953-1982 contributed positively to total factor productivity.

In this paper, we consider how change in returns to scale impacts a network of enterprises. To proceed, we specify an interesting, though small network of enterprises that encompass production processes that are interrelated through intermediacy of inputs as well as by their production of externalities. We specify the externality as controllable through its application as an intermediate input and consider two cases for its management. In the first, we allow decentralized decisions focused on profits to manage the externality while in the second we introduce an optimal tax.

The set of enterprises encompasses discrete and continuous processes that are interdependent on intermediate goods as well as their production of externalities that can be thought of as environmental effects. Response to opportunity is governed as well by adjustment costs and incomplete markets. The specification is consistent with manufacturing, crop and animal agriculture, as well as service-oriented enterprises. Potential for structural change is considered within the context of changes in returns to scale. While the model is one of optimal control, the presence of constraints opens the opportunity for timing of adjustment and constrains adjustment. The model's specification is sufficiently complex as to preempt analytical consideration. Instead, we specify a numerical example and present an illustration of the implications of the model based on numerical computation. Within this context, the dynamic implications of the evolution of scale economies is illustrated.

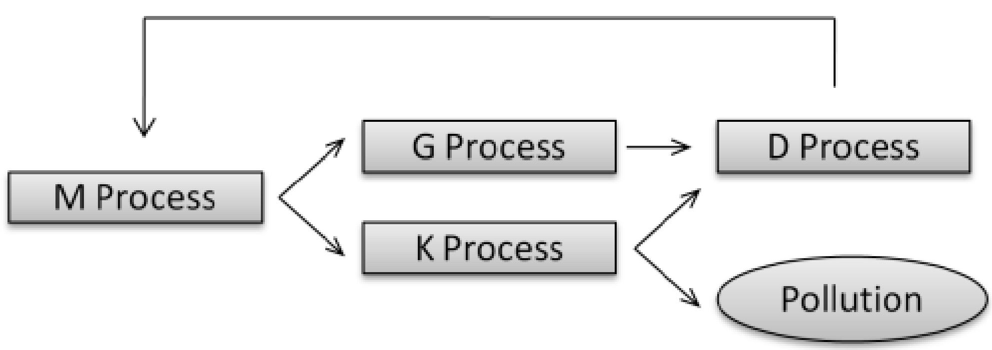
MODEL

The evolution of enterprises is considered within the context of a multiple enterprise firm that encompasses discrete and continuous processes that are interdependent on intermediate goods as well as their production of environmental effects. Response to opportunity is governed as well by adjustment costs and incomplete markets. The specification is consistent with manufacturing, crop and animal agriculture, as well as service-oriented enterprises. Potential for structural change is considered within the context of changes in prices, regulation, and technological change. We discuss management options that maximize uncertain profit over the planning horizon subject to changing price regimes, regulations, and existing and new technology. We consider incentive as well as quota type policy to regulate environmental effects. While the model is one of optimal control, the presence of constraints opens the opportunity for timing of adjustment and constrains adjustment. We derive thresholds for change and show their principal determinants. Based on this specification, we derive an indicator of flexibility as well as real options style valuation of the benefits of postponement of adjustment.

The model's specification is sufficiently complex as to preempt analytical consideration. Instead, we specify a numerical example and present illustration of the implications of the model based on numerical computation. The example illustrates both unbundling of enterprises as conditions encourage reduction in scope as well as formation of relational links to recover what might be viewed as economies-of-scale by specialized enterprises spawned by unbundling. The example also illustrates conditions necessary to induce a shift from traditional grazing animal operations integrated with crops to intensive feed lots. Within this context, shift in scale, specialization, and scope of enterprises is illustrated.

Our model incorporates four interrelated processes as described in Figure 2. In each case, nonperishable products are assumed to be stored if not sold or directly consumed as an intermediate input. Thus, we specify a continuous process (G) produces output that can be further transformed and allocated to the market, stored, or used directly as an intermediate good as an input to a continuous process (D). We suppose a batch process (K) can be implemented in any shop j of a set "shops" that occupy or utilize a limited resource (e.g. floor space, land area, ..). The shops are differentiated by their productivity. Each K process jointly produces pollution $e_{n,i}^{j}$ as well as a product $y_{k,i}^{j}$ that can be marketed, stored, or used in an intermediate input x_f to the continuous process (D). The D process is specified as producing a perishable product y_d that is instantaneously marketed and an intermediate product y_m generated by a subsidiary process (M) that may be allocated to the market, stored, or used as an intermediate input x_m to either the G or K processes. We specify the production of pollution as conditioned by the extent and efficiency of use of the intermediate product y_m in the K processes. Each process' productivity is specified as conditional on use of other inputs available from the market. As is clear from Figure 1, our conceptualization of processes allows management of pollution through dynamic changes such as storage, through intermediate product use (e.g. recycling), or through change in process operation level (e.g. shift from K to G processes). To emphasize the possibilities of internal management of pollution, we omit consideration of abatement at end-of-pipe. The inputs and outputs of each process are summarized in Table 1. As a concrete example of the enterprises considered, the K processes can be thought of as crop production, the D processes as dairy production, the G processes as hay production, and the M process as manure production. Within this context, the pollution can be interpreted as nitrogen resulting from manure or commercial fertilizer application to fields adjusted for crop uptake. Alternatively, within a manufacturing context, the K processes can be thought of batch processes, while the D and G processes are continuous processes.

FIGURE 1. INTERDEPENDENCE OF ENTERPRISES



We suppose that the batch process (K) produces $i, i = 1, 2, .., n$ where n is an integer, different outputs with timing set by we label as the "t clock" that operates in discrete time. In contrast, all other processes produce outputs on continuous time clock. The amount of output from K process producing i^{th} product in j^{th} shop at time t is defined as

$$y_{k,i}^{j}(t) = A_{k,i}^{j}(t)\left(Z_{k,i}^{j}(t)\right)^{\alpha_1}\left(x_{m,i}^{j}(t)\right)^{\alpha_2}\theta_{k,i}^{j}(t)$$

where $A_{k,i}^{j}(t)$ is the current technology's total factor productivity of the i^{th} K process in shop j, $Z_{k,i}^{j}(t)$ is an input used for i^{th} K process at shop j purchased from the outside of the firm, e.g. a part manufactured by other firms, $x_{m,i}^{j}(t)$ is an input from the inside of the firm, e.g. a part manufactured in M process, and α_1 and α_2 are parameters that translate inputs into an output. In addition, $\theta_{k,i}^{j}(t)$ enables each shop to produce only one type of K product at time t by setting $\theta_{k,i}^{j}\theta_{k,i'}^{j} = 0$, $i \neq i'$, which can be interpreted that each shop is specialized for *only* one product depending on its productivity $A_{k,i}^{j}(t)$. Therefore, it is not feasible for one shop to produce two or more types of K products at the same time. Further, we interpret $\theta_{k,i}^{j}(t)$ as indicating the scale of the shop, that is as in traditional neoclassical economics, we suppose the production process is homothetic and interpret $\theta_{k,i}^{j}(t)$ as indicating scale of operation. An alternative interpretation would be the intensity of operational use of a limited resource such as floor space. Within the context of a farm setting, $\theta_{k,i}^{j}(t)$ can be viewed as the scale of field operation for crop *j*.

To consider returns to scale within this model, we follow standard notation and define returns to scale for the jth shop operation of the K process based on the following scale function:

$$\phi(\lambda, t) = \lambda^{\alpha_1(t)+\alpha_2(t)}$$

That implies

$$\psi(t) = (\partial\phi(\lambda,t)/\partial\lambda)(\lambda/\phi(\lambda,t)) = \alpha_1(t) + \alpha_2(t)$$

To specify dynamics of returns to scale, we have introduced time dependence. To simplify, we suppose technological change is Hicks neutral and captured by $A_{k,i}^{j}(t)$. For this paper, we also limit our consideration to the case where change in returns to scale is factor neutral, i.e. $\theta_{k,i}^{j}(t)$.

$$\dot{\psi}(t) = v_1\dot{\alpha}_1(t) + v_2\dot{\alpha}_2(t) \quad where \quad \dot{\alpha}_1(t) = \dot{\alpha}_2(t) \quad and \;\; v_q = \alpha_q/(\alpha_1+\alpha_2)$$

Measurement of scale of operation follows directly from the scale function abovc. That is, for a particular value of λ^0, the scale is simply $\phi(\lambda^0, t) = \lambda^{0\ \alpha_1(t)+\alpha_2(t)}$. However, this measure focuses on scale achieved by simultaneous change in factors by the same proportion. Here, we focus on a concept of scale related to utilization of fixed production capacity as reflected by $\theta_{k,i}^{j}(t)$.

We suppose that technical change has dynamics over time as a result of two processes that are controllable by the firm. That is, we write $\dot{A}^{j}_{k,i}(t) = dA^{j}_{k,i}(t)/dt$ as

$$\dot{A}^{j}_{k,i}(t) = a_0 x^{j}_{m,i}(t) A^{j}_{k,i}(t) - a_y y^{j}_{k,i}(t)$$

where a_0 and a_y are parameters. We suppose that the first term indicates amelioration of productivity through application of the intermediate input $x^{j}_{m,i}(t)$, conditional on current productivity $A^{j}_{k,i}(t)$, while the second term indicates depreciation of productivity conditioned on output $y^{j}_{k,i}(t)$. Within this notation, we note that the evolution of output from the K processes is determined by a combination of the rate of change of returns to scale, technical change, and factors of production.

We define inventory of i^{th} K product in shop j as $I^{j}_{k,i}(t)$. Inventory dynamics are defined by $\dot{I}^{j}_{k,i}(t) = dI^{j}_{k,i}(t)/dt$ as

$$\dot{I}^{j}_{k,i}(t) = y^{j}_{k,i}(t) - s^{j}_{k,i}(t) - x^{j}_{k,i}(t)$$

where $s^{j}_{k,i}(t)$ is quantity marketed or sales of i^{th} K product at shop j. Note that the summation of all internal inputs used for K process must be less than or equal to the total amount of inventory in M process which will be introduced.

$$x_m(t) \equiv \sum_i \sum_j x^{j}_{m,i}(t)$$
$$x_m(t) \le \sum_m I_m(t)$$

Next, consider the D process. We assume that production is continuous labeling its clock as the "τ clock". The output of D process is specified as perishable. We further assume that there are vintages on machines involved in the D process that differ in their productivity. We define the D process as:

$$y_d(\tau) \equiv L_{d,}(\tau) Z_d^{\xi_1}(\tau) x_f^{\xi_2}(\tau)$$
$$\xi_1 + \xi_2 \le 1$$

where ξ_1 and ξ_2 are parameters. Recall that the output from D process is perishable, we specify $y_d(\tau)$ as nonstorable. $L_d(\tau)$ is defined as a vintage function defined as follows:

$$L_d(\tau) \equiv \mu v_d^{\omega_1}(\tau) e^{\omega_2 v_d}$$
$$v_d(\tau) = \tau - \tau_d$$

where ω_1 and ω_2 are parameters and τ_d is the time at which machine d is introduced for the first time. We specify $L_d(\tau)$ as a nonlinear productivity function. We assume machines have the highest productivity just after a warm up period, after which productivity decreases with time. M process is a by-product of D process with output $y_m(\tau)$ defined as

$$y_m(\tau) \equiv H * Z_m^{\beta_1}(\tau) * x_f^{\beta_2}(\tau),\ \beta_1 + \beta_2 \leq 1$$

where β_1 and β_2 are parameters, H is a productivity factor, and $Z_m(\tau)$ is an input to M process purchased from the outside of the firm. The inventory of M process is denoted as $I_m(\tau)$ and we write $\dot{I}_m(\tau) = dI_m(\tau)/dt$ as

$$\dot{I}_m(\tau) = y_m(\tau) - s_m(\tau) - x_m(\tau)$$

In G process, the production process is continuous on the c clock and the output produced in G process is a non-perishable and can be allocated to two distinct marketable outputs. The output of G process is defined as

$$y_g(\tau) = A_g(\tau) * (Z_g(\tau))^{\gamma_z} * (x_m(\tau))^{\gamma_x}$$

where γ_z and γ_x are parameters. $A_g(\tau)$ is a productivity function which is defined as

$$\dot{A}_g(\tau) = a_{g,0} x_m(\tau) A_g(\tau) - a_{g,y} y_g(\tau)$$

where $a_{g,0}$ and $\alpha_{g,y}$ are parameters. The inventory of G process is $I_g(\tau)$ and we write $\dot{I}_g(\tau) = \ dI_g(\tau)/dt$ as

$$\dot{I}_g(\tau) = y_g(\tau) - s_g(\tau) - x_g(\tau) - y_c(\tau)$$

where $s_g(\tau)$ is the sale of a perishable form of the output from G process and $y_c(\tau)$ is the volume of a nonperishable form defined as

$$y_c(\tau) \equiv \dot{I}_c(\tau) + s_c(\tau) + x_c(\tau)$$

where $s_c(\tau)$ is a sale of nonperishable form of output from G process.

Next, we specify the pollution process as dependent on a joint product of the K process defined as $y_{n,i}^{j}$:

$$y^{j}{}_{n,i}(\tau) \equiv n_f\left(x^{j}{}_{m}(t)+1\right)\left(x^{j}{}_{f}(\tau)+1\right)+n_k\left(\sum_j Z_{k,i}^{j}(t)\right)$$

where n_f and n_k are parameters. Here, the pollution is specified as controllable through choice of intermediate and commercial inputs. We define $x_f(\tau)$ as a result of intermediate input use by the D process:

$$x_f(\tau) \equiv \lambda_k \sum_i x_{k,i}(t) + \lambda_g x_g(\tau) + \lambda_c x_c(\tau)$$

where λ_k, λ_g, and λ_c are parameters. We define $e_{n,i}(\tau)$ as a stock of pollution generated by a combination of production and usage, e.g. by recycling:

$$\frac{de_{n,i}(\tau)}{d\tau} \equiv y_{n,i}(\tau) - u_{k,i}(\tau) - b_{n,i}(\tau), \quad i = 1, 2, \ldots, n$$

where $b_{n,i}(\tau)$ is a stock abatement at time τ and $u_{k,i}(\tau)$ is the amount of the recycle of by-products. Let $k\left(b_{n,i}(\tau), e_n(\tau)\right)$ denote the stock abatement cost, which we define as

$$k\left(b_{n,i}(\tau), e_n(\tau)\right) = c_b b_{n,i}^{\eta_1}(\tau) + c_e e_n^{\eta_2}, \quad \eta_1 + \eta_2 \leq 1$$

The amount of recycling is stated as

$$u_{k,i}(\tau) = u_{kk} * v_k(\tau) * \sum_j y_{k,i}^{j}(t)$$

$$v_k(\tau) = 1/\tau^2$$

where u_{kk} is a parameter, $v_k(t)$ is a deterioration function. The amount of recycling of by-product is proportional to the amount of K process output and it deteriorates over time. We assume that the cost of recycle is proportional to the amount of it, which can be stated as $c_u u_{k,i}(\tau)$ where c_u is the coefficient.

Table 1: Inputs and outputs of each process.

Process	Input	Output	Sell	Intermediate	Inventory
K	$x^j_{m,i}, Z^j_{k.i}$	$y^j_{k,i}$	$s^j_{k,i}$	x_k	$I^j_{k,i}$
M	x_f, Z_m	y_m	s_m	x_m	I_m
D	x_f, Z_d	y_d	N/A	N/A	N/A
N	x_f, x_m, Z^j_k	y_n	e^j_n (by-products)	N/A	I_n
G	x_m, Z_g	y_g, y_c	s_c	x_g, x_c	I_g, I_c

Based on these production processes, we define aggregate profits $\pi(\tau)$ as

$$\begin{aligned}\pi(\tau) \equiv & \sum_t \sum_i P_k s_{k,i}(t) + \int_{\tau_0}^{\tau_f} \sum_m P_m s_m(\tau) d\tau + \int_{\tau_0}^{\tau_f} \sum_d P_d s_d(\tau) d\tau \\ & + \int_{\tau_0}^{\tau_f} \sum_c P_c s_c(\tau) d\tau - \sum_t \sum_k R_k Z_k(t) - \int_{\tau_0}^{\tau_f} \sum_m R_m Z_m(\tau) d\tau \\ & - \int_{\tau_0}^{\tau_f} \sum_d R_d Z_d(\tau) d\tau - \int_{\tau_0}^{\tau_f} \sum_k c_k I_k^{c_1}(\tau) d\tau - \int_{\tau_0}^{\tau_f} \sum_m c_m I_m^{c_2}(\tau) d\tau \\ & - \int_{\tau_0}^{\tau_f} \sum_n c_n I_n(\tau) d\tau - \int_{\tau_0}^{\tau_f} \sum_g c_c I_c(\tau) d\tau - \int_{\tau_0}^{\tau_f} \sum_i k\left(b_{n,i}(\tau), e_n(\tau)\right) \\ & - \int_{\tau_0}^{\tau_f} \sum_i c_u u_{k,i}(\tau) - \int_{\tau_0}^{\tau_f} \sum_n P_n e_n(\tau) d\tau\end{aligned}$$

To proceed, we consider the growth dynamics of this system. To do so, we suppose the system's enterprises operate such that aggregate profits are maximized in an optimal control problem. In the absence of central regulation of the pollution, the enterprises find individual interests in managing pollution conditional on the intensity of their use of y_n as an intermediate product. In order to consider dynamics associated with centralized regulation of pollution we define the aggregate objective as considering both profits and the aggregate level of pollution. That is, we define:

$$w(\tau) \equiv w_1 e_n(\tau) + w_2 \left(e_n(\tau)\right)^2 + w_3 \pi(\tau)$$

However, to retain focus on decentralized enterprises, we consider cases where *w1=w2 =0 and w3=1*. Thus, we suppose decentralized management solves the following aggregate control problem:

$$J = \max \int_{\tau_0}^{\tau_f} w(\tau) d\tau$$

NUMERICAL RESULTS

To provide an illustration, we solve the corresponding mathematical program using a discrete time approximation with N = 20 equal time steps. In these illustrations, we implement optimization using GAMS/MINOS.

To proceed, we consider the implications of returns to scale in the K process through a set of numerical simulations. Specification of values for parameters and exogenous variables such as market prices are presented in the appendix. We consider a 20 period planning horizon. We consider the following cases:

CASE 1: Near Constant returns to scale, $\psi(t) = 0.99$.
CASE 2: Decreasing returns to scale $\psi(t) = 0.80$, no abatement.
CASE 3: Decreasing returns to scale $\psi(t) = 0.50$, no abatement.

In each of these cases, recall the K process is specified as having declining productivity and is operated as a batch process only at discrete times, here specified as multiples of 5.

CASE 1: Near Constant returns to scale, $\psi(t) = 0.99$.
Figure 2 shows case 1a results. The upper left panel of Figure 2 illustrates output of K process where it is clear that it is optimal to specialize in production of only one output, however the nature of that specialization varies over time. In upper right panel of Figure 2, we see output of M and D process. Recall, for the D process we specified two operations based on differing vintages (e.g. cow lactation, or machine use). As these are continuous processes conditional on the discrete time K processes we note what can be interpreted as seasonality. The lower left panel of Figure 2 shows output of the discrete time G process. In Figure 2, lower right panel, we see pollution fluctuates with the operation of the K process that uses $y_{n,i}^{j}$ as an intermediate input. In Figure 3, we present the dynamics of scale of operation based on optimal paths found for the scale measures, $\theta_{k,i}^{j}(t)$. The results clearly indicate that it is optimal to specialize K process production in particular shops. For example, in the first and second time periods (*t=5, 10*), only product 3 is produced and only in shop 2. In time period 15, only product 1 is produced and only in shop 1, while in time period 20, only product 2 is produced and only in shop 2. This specialization allows for control of pollution while meeting demands for intermediate input use of the K process outputs.

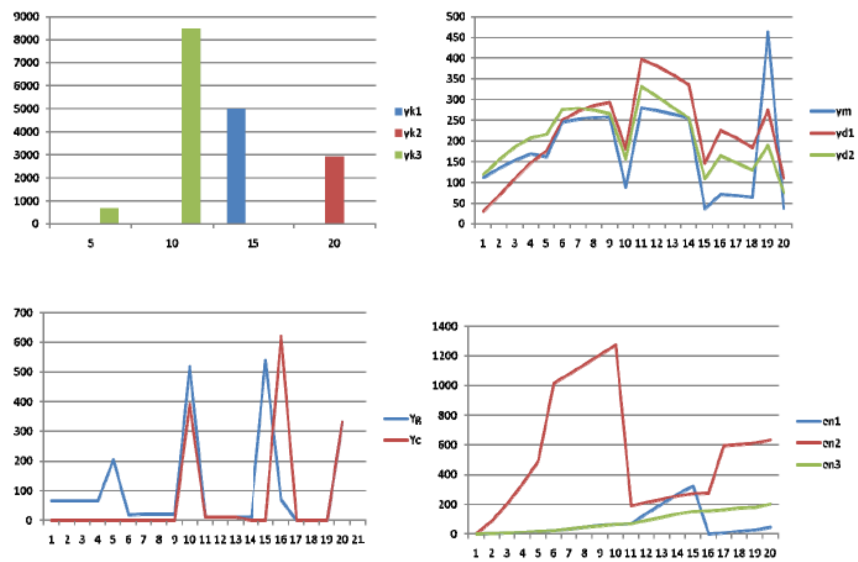


FIGURE 2. Case 1a. Output and Pollution Dynamics

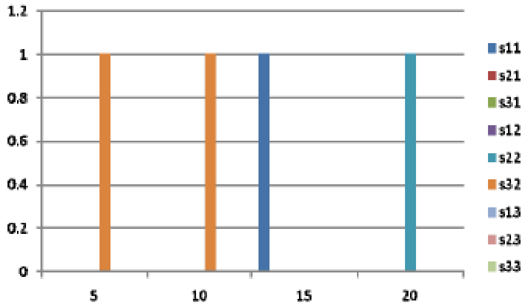


FIGURE 3. Case 1a Optimal scales of operation
$\theta_{k,i}^{j}(t)$ = sij = scale of i^{th} product by j^{th} shop.

CASE 2: Decreasing returns to scale $\psi(t) = 0.80$.
Here we see diversification in outputs in each time period, see Figure 4, as well as across shops, see Figure 5. We see from Figure 5 that in the first and second production periods, product 1 is produced in shops 1 and 2, and product 2 in ship 3. In the third time period, shop 1 produces product 2 and shop 3 produces product 3. In the forth time period, shop 1 continues producing product 2, while shop 2 produces product 1.

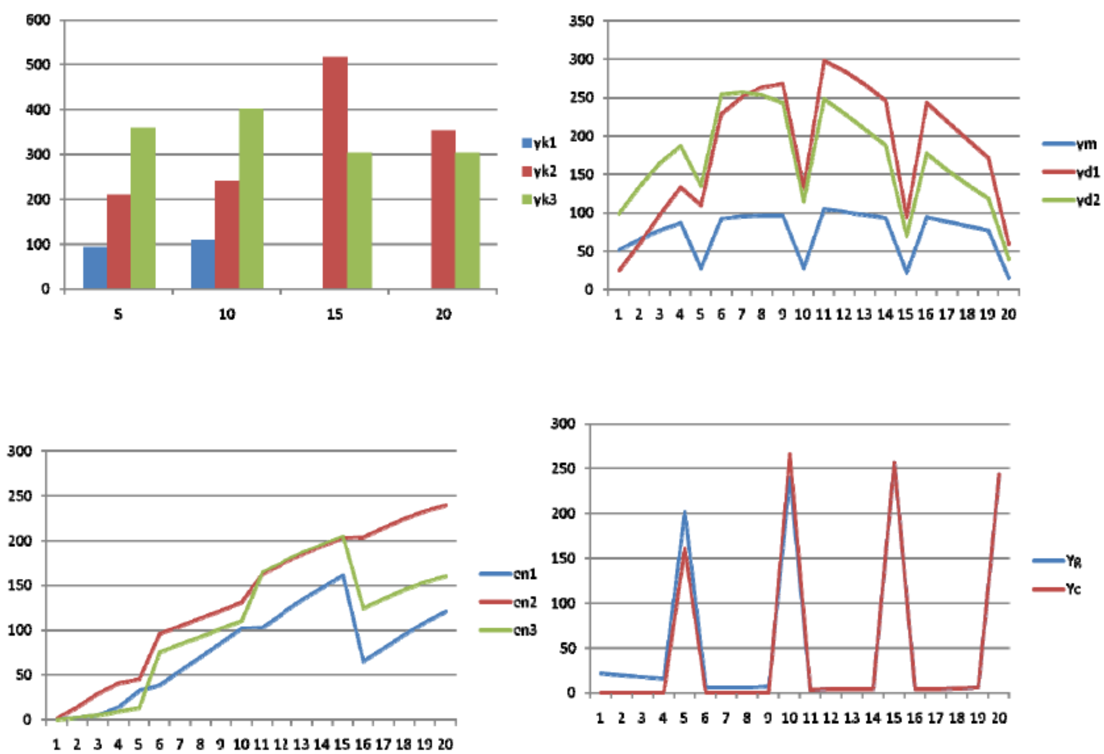


FIGURE 4 CASE 2: Decreasing returns to scale $\psi(t) = 0.80$.

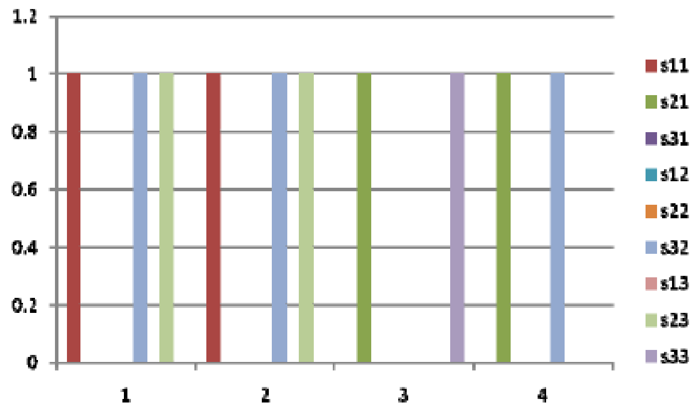


FIGURE 5 CASE 2: Decreasing returns to scale *ψ(t) = 0.80*.
Optimal scales of operation $\theta_{k,i}^{j}(t)$ = sij = scale of i^{th} product by j^{th} shop.

CASE 3: Decreasing returns to scale *ψ(t) = 0.50*.
Here, in comparison to Case 1 we see a dramatic change in the structure of production. First, in the upper left panel we see a diversification of outputs from the K process. This diversification is also seen in Figure 6 where in contrast to the specialization seen in Figure 4, for this case we see that multiple shops are operated each time period. Here, we see for the first production period, output 1 is produced only in shop 1, while output 3 is produced in both shop 2 and 3. In the second and the third period, output 1 is produced only in shop 1, while output 2 is produced in shop 2, output 3 in shop 3. In the forth period, output 2 is produced in shop 1 and shop 2, while output 3 is produced in shop 3.

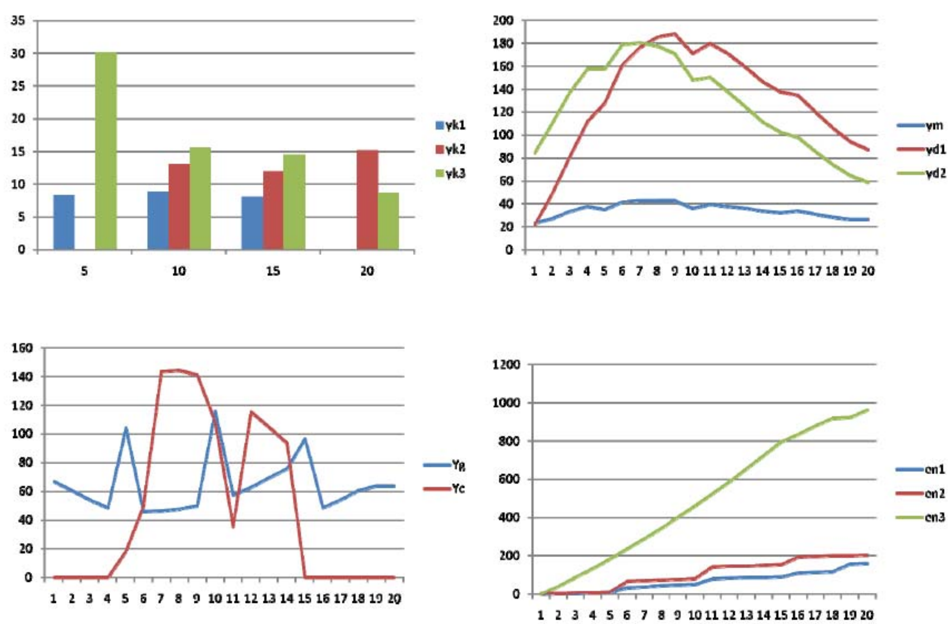


FIGURE 6 CASE 2 Decreasing returns to scale *ψ(t) = 0.50*.

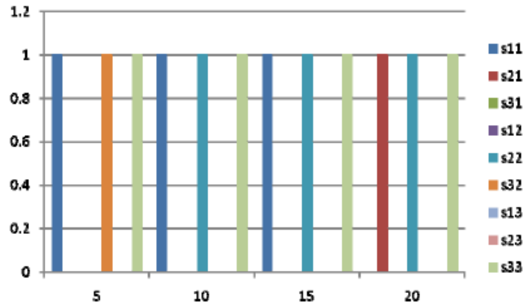


FIGURE 7 CASE 2 Decreasing returns to scale $\psi(t) = 0.50$.
Optimal scales of operation $\theta^{j}_{k,i}(t)$ = sij = scale of i^{th} product by j^{th} shop.

Interpretation of these dynamics with respect to changes in returns to scale is as follows. Under near constant returns to scale (Case 1), each product is produced only in one production period, and in only one shop. This can be thought of production at scale 1. We find similar scale of production, though in the second through forth periods, scale is expanded to 2 for output 2. That is, scale of production of output 2 is doubled under a dramatic reduction in returns to scale. This is intuitive if we view the expression of returns to scale in the curvature of an enterprise's average cost curve. A reduction in returns to scale at any point along the average cost curve follows from an increase in the degree of curvature of that curve. That is, it shifts from a shallow cup to one with steeper sides. It follows that the optimal output is reduced.

CONCLUSIONS AND FUTURE DIRECTIONS

Results presented here provide preliminary illustrations of the dynamic implications of discrete changes in returns to scale. We limited our focus here to short-run dynamics. Within this dynamics context, the implications of changes in returns to scale are complicated and obscured by multiple output choices and interactions across processes. Results presented here were generated over a relatively short time horizon, in further research the time horizon will be extended to examine steady state, stability, and cyclic implications of changes in returns to scale. Nonetheless, several conclusions appear to be supportable. First, as returns to scale increase for one of a set of interrelated technologies, we see increased specialization. Second, for the specified interrelationships in our model, we see that as returns increase, the growth of the pollution stock is reduced.

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