Measurement of dynamic efficiency, a directional distance function parametric approach

Teresa Serra[1], Spiro Stefanou[2,3] and Alfons Oude Lansink[3]

[1]Centre de Recerca en Economia i Desenvolupament Agroalimentari (CREDA-UPC-IRTA) Spain, teresa.serra-devesa@upc.edu; [2]Department of Agricultural Economics and Rural Sociology, Penn State University, USA; [3]Business Economics Group, Wageningen University, Netherlands.

Paper prepared for presentation at the 114th EAAE Seminar 'Structural Change in Agriculture', Berlin, Germany, April 15 - 16, 2010

Abstract

This research proposes a parametric estimation of the structural dynamic efficiency measures proposed by Silva and Oude Lansink (2009). Overall, technical and allocative efficiency measurements are derived based on a directional distance function and the duality between this function and the optimal value function. The applicability of the parametric proposal is illustrated by assessing dynamic efficiency ratings for a sample of Dutch dairy farms observed from 1995 to 2005.

Keywords: structural dynamic efficiency, dairy farms, parametric approach

JEL classification: D21, D24, D61, D92.

1. Introduction

The economics literature on efficiency has traditionally derived static technical efficiency measures that ignore the adjustment of quasi-fixed inputs to their long-run levels and the time interdependence of production decisions. Only recently have we witnessed important contributions to the literature on dynamic efficiency modeling. In this regard, it is noteworthy that most of the advances have taken place in the framework of the nonparametric data envelopment analysis (DEA). While Sengupta (1995) introduced the first order conditions of the dynamic optimization problem into the DEA models, Nemoto and Goto (1999, 2003) considered the stock of capital at the end of a time period as an additional output within the DEA model. Silva and Stefanou (2007) proposed nonparametric measures of dynamic efficiency based on Silva and Stefanou's (2003) nonparametric dynamic dual cost approach to production analysis. More recently, Silva and Oude Lansink (2009) have employed the adjustment cost technology to generalize the static conditional input distance function developed by Chambers et al. (1998) to a dynamic framework. The empirical application of their proposal is illustrated using DEA methods.

While a number of parametric reduced-form approaches to dynamic efficiency measurement have been proposed (Tsionas, 2006; Ahn and Sickles, 2000), structural approaches have been very scarce. The paper by Rungsuriyawiboon and Stefanou (2007) is a notable exception. These authors propose a shadow cost approach in the framework of the dynamic duality model of intertemporal decision making to generate both allocative and technical efficiency measures. In being based on the dynamic duality theory of intertemporal decision making, the approach by these authors does not however specify nor estimate the production technology. The proposal by Silva and Oude Lansink (2009) generates efficiency measures based on the production technology. The duality between this function and the optimal value function is developed and allocative efficiency measures are subsequently derived. Silva and Oude Lansink's (2009) method is of particular interest over previous proposals of dynamic efficiency measurement, since the technology is specified as a directional distance function. Directional distance functions are a more general and less restrictive specification of technology than traditional specifications of the production frontier. Our work contributes to previous literature by parametrically estimating the model proposed by Silva and Oude Lansink (2009). As has been noted above, while nonparametric methods have been shown to be an adequate methodology to measure dynamic efficiency, structural parametric applications have been very scarce, making the analysis of this issue necessary.

2. The dynamic directional distance function, the intertemporal optimization problem and duality

Following Silva and Oude Lansink (2009), a directional distance function is used to generate farm-level dynamic technical inefficiency measures for all factors of production. Let $\mathbf{y} \in \Re_{++}^{M}$ represent a vector of outputs, $\mathbf{x} \in \Re_{+}^{N}$ denote a vector of variable inputs, $\mathbf{K} \in \Re_{++}^{F}$ the capital stock vector, $\mathbf{I} \in \Re_{+}^{F}$ the vector of gross investments and $\mathbf{L} \in \Re_{++}^{C}$ a vector of fixed inputs for which no investments are allowed. The production input requirement set can be represented as $V(\mathbf{y}:\mathbf{K},\mathbf{L}) = \{(\mathbf{x},\mathbf{I}) : (\mathbf{x},\mathbf{I}) \text{ can produce } \mathbf{y} \text{ given } \mathbf{K},\mathbf{L}\}$. The input requirement set is assumed to have the properties defined by Silva and Oude Lansink (2009), i.e., $V(\mathbf{y}:\mathbf{K},\mathbf{L})$ is a closed and nonempty set, has a lower bound, is positive monotonic in $\mathbf{x}$, negative monotonic in $\mathbf{I}$, is a strictly convex set, output levels increase with the stock of capital and quasi-fixed inputs and can be disposed of freely.

The input-oriented dynamic directional distance function $\vec{D}^i(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{x},\mathbf{I};\mathbf{g_x},\mathbf{g_I})$ can be defined as follows:

$$\vec{D}^i(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{x},\mathbf{I};\mathbf{g_x},\mathbf{g_I}) = \max\left\{\beta \in \Re : \left(\mathbf{x} - \beta\mathbf{g_x}, \mathbf{I} + \beta\mathbf{g_I}\right) \in V(\mathbf{y}:\mathbf{K},\mathbf{L})\right\}, \quad \mathbf{g_x} \in \Re^N_{++},\ \mathbf{g_I} \in \Re^F_{++}, \left(\mathbf{g_x},\mathbf{g_I}\right) \neq \left(\mathbf{0}^N, \mathbf{0}^F\right) \tag{1}$$

if $\left(\mathbf{x} - \beta\mathbf{g_x}, \mathbf{I} + \beta\mathbf{g_I}\right) \in V(\mathbf{y}:\mathbf{K},\mathbf{L})$ for some β, $\vec{D}^i(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{x},\mathbf{I};\mathbf{g_x},\mathbf{g_I}) = -\infty$, otherwise.

The distance function is a measure of the maximal translation of $(\mathbf{x},\mathbf{I})$ in the direction defined by the vector $\left(\mathbf{g_x},\mathbf{g_I}\right)$, that keeps the translated input combination inside $V(\mathbf{y}:\mathbf{K},\mathbf{L})$. Since $\beta\mathbf{g_x}$ is substracted from $\mathbf{x}$ and $\beta\mathbf{g_I}$ is added to $\mathbf{I}$, the directional distance function is defined by simultaneously contracting variable inputs and expanding gross investments. As shown by Silva and Oude Lansink (2009), $\vec{D}^i(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{x},\mathbf{I};\mathbf{g_x},\mathbf{g_I}) \geq 0$ fully characterizes the input requirement set $V(\mathbf{y}:\mathbf{K},\mathbf{L})$, being thus an alternative primal representation of the adjustment cost production technology.

The input-oriented dynamic directional distance function inherits the properties of the static directional input function. These properties are:

D.1. If $V(\mathbf{y}:\mathbf{K},\mathbf{L})$ is strictly convex, then $\vec{D}^i(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{x},\mathbf{I};\mathbf{g_x},\mathbf{g_I})$ is strictly concave with respect to $(\mathbf{x},\mathbf{I})$ given $\mathbf{y}$, $\mathbf{K}$ and $\mathbf{L}$.

D.2. $\vec{D}^i(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{x} - \alpha\mathbf{g_x},\mathbf{I} + \alpha\mathbf{g_I};\mathbf{g_x},\mathbf{g_I}) = \vec{D}^i(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{x},\mathbf{I};\mathbf{g_x},\mathbf{g_I}) - \alpha$, $\alpha \in \Re$.

D.3. If outputs can be disposed of freely, then $\mathbf{y'} \geq \mathbf{y} \Rightarrow \vec{D}^i(\mathbf{y'}, \mathbf{K}, \mathbf{L}, \mathbf{x}, \mathbf{I}; \mathbf{g_x}, \mathbf{g_I}) < \vec{D}^i(\mathbf{y}, \mathbf{K}, \mathbf{L}, \mathbf{x}, \mathbf{I}; \mathbf{g_x}, \mathbf{g_I})$.

D.4. If $V(\mathbf{y} : \mathbf{K}, \mathbf{L})$ is positive monotonic in $\mathbf{x}$, then $\mathbf{x'} \geq \mathbf{x} \Rightarrow \vec{D}^i(\mathbf{y}, \mathbf{K}, \mathbf{L}, \mathbf{x'}, \mathbf{I}; \mathbf{g_x}, \mathbf{g_I}) > \vec{D}^i(\mathbf{y}, \mathbf{K}, \mathbf{L}, \mathbf{x}, \mathbf{I}; \mathbf{g_x}, \mathbf{g_I})$.

D.5. If $V(\mathbf{y} : \mathbf{K}, \mathbf{L})$ is negative monotonic in $\mathbf{I}$, then $\mathbf{I'} \leq \mathbf{I} \Rightarrow \vec{D}^i(\mathbf{y}, \mathbf{K}, \mathbf{L}, \mathbf{x}, \mathbf{I'}; \mathbf{g_x}, \mathbf{g_I}) > \vec{D}^i(\mathbf{y}, \mathbf{K}, \mathbf{L}, \mathbf{x}, \mathbf{I}; \mathbf{g_x}, \mathbf{g_I})$.

D.6. If output levels are increasing in the stock of capital, then $\mathbf{K'} \geq \mathbf{K} \Rightarrow \vec{D}^i(\mathbf{y}, \mathbf{K'}, \mathbf{L}, \mathbf{x}, \mathbf{I}; \mathbf{g_x}, \mathbf{g_I}) > \vec{D}^i(\mathbf{y}, \mathbf{K}, \mathbf{L}, \mathbf{x}, \mathbf{I}; \mathbf{g_x}, \mathbf{g_I})$.

D.7. If output levels are increasing in the stock of fixed inputs, then $\mathbf{L'} \geq \mathbf{L} \Rightarrow \vec{D}^i(\mathbf{y}, \mathbf{K}, \mathbf{L'}, \mathbf{x}, \mathbf{I}; \mathbf{g_x}, \mathbf{g_I}) > \vec{D}^i(\mathbf{y}, \mathbf{K}, \mathbf{L}, \mathbf{x}, \mathbf{I}; \mathbf{g_x}, \mathbf{g_I})$.

D.8. $\vec{D}^i(\mathbf{y}, \mathbf{K}, \mathbf{L}, \mathbf{x}, \mathbf{I}; \alpha\mathbf{g_x}, \alpha\mathbf{g_I}) > \dfrac{1}{\alpha}\vec{D}^i(\mathbf{y}, \mathbf{K}, \mathbf{L}, \mathbf{x}, \mathbf{I}; \mathbf{g_x}, \mathbf{g_I})$, $\alpha > 0$.

D.9. $\vec{D}^i(\mathbf{y}, \mathbf{K}, \mathbf{L}, \mathbf{x}, \mathbf{I}; \mathbf{g_x}, \mathbf{g_I})$ is continuous with respect to $(\mathbf{x}, \mathbf{I})$, given $\mathbf{K}$, $\mathbf{L}$ and $\mathbf{y}$.

It is assumed that firms are intertemporally cost minimizing and thus they take their decisions in accord with the following optimization problem:

$$W(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{w},\mathbf{c}) = \min_{\mathbf{x},\mathbf{I}} \int_t^{\infty} e^{-rt} \left[\mathbf{w'x} + \mathbf{c'K}\right] dt$$

$$s.t. \qquad (2)$$

$$\dot{\mathbf{K}} = \mathbf{I} - \boldsymbol{\delta}\mathbf{K}$$

$$\vec{D}^i(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{x},\mathbf{I};\mathbf{g}_\mathbf{x},\mathbf{g}_\mathbf{I}) \geq 0$$

where $\mathbf{w} \in \mathbf{R}^{\mathbf{N}}_{++}$ is a variable input price vector, $\mathbf{c} \in \mathbf{R}^{\mathbf{F}}_{++}$ is a vector of capital rental prices, $\boldsymbol{\delta}$ is a diagonal matrix containing depreciation rates and r is the discount rate.

Chambers et al. (1998) establish duality between static directional input distance functions and the static cost function. Silva and Oude Lansink (2009) prove that $\vec{D}^i(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{x},\mathbf{I};\mathbf{g}_\mathbf{x},\mathbf{g}_\mathbf{I})$ is dual to $W(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{w},\mathbf{c})$. Dynamic duality is based on the dynamic input distance function properties defined above (see Silva and Oude Lansink, 2009 for further detail). The Hamilton-Jacobi-Bellman (H-J-B) equation corresponding to the optimization program can be expressed as:

$$rW(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{w},\mathbf{c}) = \min_{\mathbf{x},\mathbf{I}} \left\{\mathbf{w'x} + \mathbf{c'K} + W_\mathbf{k}(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{w},\mathbf{c})'(\mathbf{I} - \boldsymbol{\delta}\mathbf{K}) + \lambda\vec{D}^i(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{x},\mathbf{I};\mathbf{g}_\mathbf{x},\mathbf{g}_\mathbf{I})\right\} \qquad (3)$$

Where $W_\mathbf{k}(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{w},\mathbf{c})$ is the first derivative of $W(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{w},\mathbf{c})$ with respect to $\mathbf{K}$ and $\lambda = W_\mathbf{k}(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{w},\mathbf{c})\mathbf{g}_\mathbf{I} - \mathbf{w'g}_\mathbf{x}$. From the H-J-B equation in (3), the duality between $\vec{D}^i(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{x},\mathbf{I};\mathbf{g}_\mathbf{x},\mathbf{g}_\mathbf{I})$ and $W(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{w},\mathbf{c})$ is given by the following optimization problems:

$$rW(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{w},\mathbf{c}) = \min_{\mathbf{x},\mathbf{I}} \{\mathbf{w'x} + \mathbf{c'K} + W_\mathbf{k}(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{w},\mathbf{c})'(\mathbf{I} - \boldsymbol{\delta}\mathbf{K}) +$$
$$(W_\mathbf{k}(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{w},\mathbf{c})\mathbf{g}_\mathbf{I} - \mathbf{w'g}_\mathbf{x})\vec{D}^i(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{x},\mathbf{I};\mathbf{g}_\mathbf{x},\mathbf{g}_\mathbf{I})\} \qquad (4a)$$

$$\vec{D}^i(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{x},\mathbf{I};\mathbf{g}_\mathbf{x},\mathbf{g}_\mathbf{I}) = \min_{\mathbf{w},\mathbf{c}} \left\{ \frac{\mathbf{w'x} + \mathbf{c'K} + W_\mathbf{k}(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{w},\mathbf{c})'(\mathbf{I} - \boldsymbol{\delta}\mathbf{K}) - rW(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{w},\mathbf{c})}{\mathbf{w'g_x} - W_\mathbf{k}(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{w},\mathbf{c})\mathbf{g_I}} \right\} \quad (4b)$$

From the previous optimization problems, Silva and Oude Lansink (2009) derive a dynamic inefficiency measurement. The dynamic cost inefficiency can be expressed as:

$$OI^i = \frac{\mathbf{w'x} + \mathbf{c'K} + W_\mathbf{k}(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{w},\mathbf{c})'(\mathbf{I} - \boldsymbol{\delta}\mathbf{K}) - rW(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{w},\mathbf{c})}{\mathbf{w'g_x} - W_\mathbf{k}(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{w},\mathbf{c})\mathbf{g_I}} \geq \vec{D}^i(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{x},\mathbf{I};\mathbf{g}_\mathbf{x},\mathbf{g}_\mathbf{I}) \quad (5)$$

OI^i is the difference between the shadow cost of actual input choices and the minimum shadow cost, normalized by the shadow value of the direction vector. $OI^i \geq \vec{D}^i(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{x},\mathbf{I};\mathbf{g}_\mathbf{x},\mathbf{g}_\mathbf{I})$, being $\vec{D}^i(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{x},\mathbf{I};\mathbf{g}_\mathbf{x},\mathbf{g}_\mathbf{I})$ a measure of technical inefficiency (TI^i) of both variable and quasi-fixed inputs. The difference between the dynamic cost and technical inefficiencies yields the allocative inefficiency ($AI^i \geq 0$):

$$OI^i = \vec{D}^i(\mathbf{y},\mathbf{K},\mathbf{L},\mathbf{x},\mathbf{I};\mathbf{g}_\mathbf{x},\mathbf{g}_\mathbf{I}) + AI^i \quad (6)$$

In the next section, we present the empirical specification of both the directional distance function and the minimum shadow cost function. Estimation methods are also discussed.

3. Empirical specification

Following Chambers (2002) and Färe et al. (2005), the quadratic function is used as a parametric specification for the directional distance function. The quadratic specification offers the advantage that it can be easily restricted to satisfy property D.2., the so called translation property. If we set $g_{xi}=1,\ i=1,...,N$, $g_{Ij}=1,\ j=1,...,F$, $M=1$ (i.e., we assume a single-output firm) and add a time trend (t), the distance function for the firm h can be expressed as:

$$\begin{aligned}
\vec{D}_h^i(y,\mathbf{K},\mathbf{L},\mathbf{x},\mathbf{I},t;1,1) &= a + a_y y + \sum_{n=1}^{C} a_{Ln} L_n + \sum_{j=1}^{F} a_{Ij} I_j + \sum_{i=1}^{N} a_{xi} x_i + \sum_{j=1}^{F} a_{Kj} K_j + \\
&\frac{1}{2} a_{yy} y^2 + \frac{1}{2}\sum_{n=1}^{C}\sum_{n'=1}^{C} a_{LnLn'} L_n L_{n'} + \frac{1}{2}\sum_{j=1}^{F}\sum_{j'=1}^{F} a_{IjIj'} I_j I_{j'} + \frac{1}{2}\sum_{i=1}^{N}\sum_{i'=1}^{N} a_{xixi'} x_i x_{i'} + \frac{1}{2}\sum_{j=1}^{F}\sum_{j'=1}^{F} a_{KjKj'} K_j K_{j'} + \\
&\sum_{n=1}^{C} a_{yLn} y L_n + \sum_{j=1}^{F} a_{yIj} y I_j + \sum_{i=1}^{N} a_{yxi} y x_i + \sum_{j=1}^{F} a_{yKj} y K_j + \sum_{n=1}^{C}\sum_{j=1}^{F} a_{LnIj} L_n I_j + \sum_{n=1}^{C}\sum_{i=1}^{N} a_{Lnxi} L_n x_i + \\
&\sum_{n=1}^{C}\sum_{j=1}^{F} a_{Lnkj} L_n K_j + \sum_{j=1}^{F}\sum_{i=1}^{N} a_{Ijxi} I_j x_i + \sum_{j=1}^{F}\sum_{j'=1}^{F} a_{IjKj'} I_j K_{j'} + \sum_{j=1}^{F}\sum_{i=1}^{N} a_{Kjxi} K_j x_i + a_t t
\end{aligned} \tag{7}$$

Parameter restrictions that need to be imposed for the translation property to hold are:

$\sum_{j=1}^{F} a_{Ij} - \sum_{i=1}^{N} a_{xi} = -1$; $\sum_{j=1}^{F}\sum_{j'=1}^{F} a_{IjKj'} - \sum_{j=1}^{F}\sum_{i=1}^{N} a_{Kjxi} = 0$; $\sum_{j=1}^{F} a_{yIj} - \sum_{i=1}^{N} a_{yxi} = 0$;

$-\sum_{i'=1}^{N} a_{xixi'} + \sum_{j=1}^{F} a_{Ijxi} = 0,\ i=1,...,N$; $\sum_{j'=1}^{M} a_{IjIj'} - \sum_{i=1}^{N} a_{Ijxi} = 0,\ j=1,...,F$; and

$\sum_{n=1}^{C}\sum_{j=1}^{F} a_{LnIj} - \sum_{n=1}^{C}\sum_{i=1}^{N} a_{Lnxi} = 0$. Symmetry restrictions are also imposed: $a_{LnLn'} = a_{Ln'Ln}$,

$a_{IjIj'} = a_{Ij'Ij}$, $a_{xixi'} = a_{xi'xi}$, and $a_{KjKj'} = a_{Kj'Kj}$.

Following Kumbhakar and Lovell (2000) and Färe et al. (2005), the dynamic quadratic directional input distance function can be estimated using stochastic estimation techniques. The stochastic specification of the distance takes the following form:

$$0 = \vec{D}_h^i(y, \mathbf{K}, \mathbf{L}, \mathbf{x}, \mathbf{I}, t; \mathbf{1}, \mathbf{1}) + \varepsilon_h \tag{8}$$

where $\varepsilon_h = v_h - u_h$, $v_h \sim N(0, \sigma_v^2)$ and $u_h \sim N^+(0, \sigma_u^2)$. In order to estimate (8), the translation property is used:

$$-\alpha_h = \vec{D}_h^i(y, \mathbf{K}, \mathbf{L}, \mathbf{x} - \alpha_h, \mathbf{I} + \alpha_h, t; \mathbf{1}, \mathbf{1}) + \varepsilon_h \tag{9}$$

Function $\vec{D}_h^i(y, \mathbf{K}, \mathbf{L}, \mathbf{x} - \alpha_h, \mathbf{I} + \alpha_h, t; \mathbf{1}, \mathbf{1})$ corresponds to the quadratic form in (7), with α_h added to gross investments and subtracted from variable input quantities. By choosing a α_h specific for each firm, variation on the left hand side of (9) is obtained. Following Färe et al. (2005), α_h is made equal to I_1.[1]

Stochastic estimation is accomplished by maximum likelihood procedures. For a sample of H observations, the logarithm of the likelihood function is defined as:

$$L = \eta - H \ln \sigma_\varepsilon + \sum_{h=1}^{H} \ln \Phi\left(-\frac{\varepsilon_h \lambda_\varepsilon}{\sigma_\varepsilon} \right) - \frac{1}{2\sigma_\varepsilon^2} \sum_{h=1}^{H} \varepsilon_h^2 \tag{10}$$

[1] Parameter estimates changed very little with the choice of α_h however.

Where η is a constant, $\sigma_\varepsilon = \left(\sigma_u^2 + \sigma_v^2\right)^{1/2}$, $\lambda_\varepsilon = \sigma_u / \sigma_v$, and Φ is the standard normal cumulative distribution function. Point estimates of each producer's technical inefficiency can be derived as follows:

$$TI_h^i = 1 - \frac{\mathbf{X}_h\mathbf{'A} - u_h}{\mathbf{X}_h\mathbf{'A}} = 1 - TE_h^i \tag{11}$$

where $\mathbf{X}_h$ and $\mathbf{A}$ are the vectors of explanatory variables and parameter estimates respectively, TE_h^i is a measure of dynamic technical efficiency, u_h is replaced by its conditional expectation $E\left(u_h / \varepsilon_h\right) = \sigma^* \left[\frac{\phi\left(\varepsilon_h \lambda_\varepsilon / \sigma_\varepsilon\right)}{1 - \Phi\left(\varepsilon_h \lambda_\varepsilon / \sigma_\varepsilon\right)} - \frac{\varepsilon_h \lambda_\varepsilon}{\sigma_\varepsilon}\right]$, ϕ is the standard normal probability distribution function and $\sigma_*^2 = \sigma_u^2 \sigma_v^2 / \sigma_\varepsilon^2$.

Once the dynamic directional input distance function has been estimated and technical efficiency point estimates derived, one can obtain the dynamic cost inefficiency model by means of estimating the following cost frontier model, where a time trend has also been added:

$$C_h = rW(y, \mathbf{K}, \mathbf{L}, w_2, \mathbf{c}, t) - W_{\mathbf{k}}(y, \mathbf{K}, \mathbf{L}, w_2, \mathbf{c}, t)\dot{\mathbf{K}} - W_t(y, \mathbf{K}, \mathbf{L}, w_2, \mathbf{c}, t) + \xi_h \tag{12}$$

where $C_h = \frac{\mathbf{w'x} + \mathbf{c'K}}{w_1}$ is the observed long-run cost normalized by the variable input price w_1, $W(y, \mathbf{K}, \mathbf{L}, \frac{w_2}{w_1}, \frac{\mathbf{c}}{w_1}, t)$ is the optimum cost where all input prices have been normalized with respect to w_1, $W_{\mathbf{k}}(y, \mathbf{K}, \mathbf{L}, \frac{w_2}{w_1}, \frac{\mathbf{c}}{w_1}, t)$ and $W_t(y, \mathbf{K}, \mathbf{L}, \frac{w_2}{w_1}, \frac{\mathbf{c}}{w_1}, t)$ are its first derivatives with respect to $\mathbf{K}$ and t respectively, $\xi_h = \gamma_h + \delta_h$, $\gamma_h \sim N(0,\sigma_\gamma^2)$, and $\delta_h \sim N^+(0,\sigma_\delta^2)$. The cost inefficiency term δ_h corresponds to the numerator in (5). It is thus a non-normalized overall efficiency measure. By normalizing all input prices with respect to w_1, $W(.)$ is specified as:

$$\begin{aligned}
W(y,\mathbf{K},\mathbf{L},w_2,\mathbf{c},t) &= b_0 + b_y y + b_{w2}\frac{w_2}{w_1} + \sum_{j=1}^{F} b_{cj}\frac{c_j}{w_1} + \sum_{j=1}^{F} b_{kj}K_j + \sum_{n=1}^{C} b_{Ln}L_n + \\
&\frac{1}{2}b_{yy}y^2 + \frac{1}{2}b_{w2w2}\left(\frac{w_2}{w_1}\right)^2 + \frac{1}{2}\sum_{j=1}^{F}\sum_{j'=1}^{F} b_{cjcj'}\frac{c_j}{w_1}\frac{c_{j'}}{w_1} + \frac{1}{2}\sum_{j=1}^{F}\sum_{j'=1}^{F} b_{kjkj'}K_jK_{j'} + \frac{1}{2}\sum_{n=1}^{C}\sum_{n'=1}^{C} b_{LnLn'}L_nL_{n'} + \\
&b_{yw2}y\frac{w_2}{w_1} + \sum_{j=1}^{F} b_{ycj}y\frac{c_j}{w_1} + \sum_{j=1}^{F} b_{ykj}yK_j + \sum_{n=1}^{C} b_{yLn}yL_n + \sum_{j=1}^{F} b_{w2cj}\frac{w_2}{w_1}\frac{c_j}{w_1} + \sum_{j=1}^{F} b_{w2kj}\frac{w_2}{w_1}K_j + \\
&\sum_{n=1}^{C} b_{w2Ln}\frac{w_2}{w_1}L_n + \sum_{j=1}^{F}\sum_{j'=1}^{F} b_{kjcj'}K_j\frac{c_{j'}}{w_1} + \sum_{j=1}^{F}\sum_{n=1}^{C} b_{cjLn}\frac{c_j}{w_1}L_n + \sum_{j=1}^{F}\sum_{n=1}^{C} b_{kjLn}K_jL_n + b_t t
\end{aligned} \qquad (13)$$

Symmetry restrictions $b_{cjcj'} = b_{cj'cj}$, $b_{kjkj'} = b_{kj'kj}$, $b_{LnLn'} = b_{Ln'Ln}$, and $b_{kjcj'} = b_{kj'cj}$ are imposed so as to make the model more tractable.

Given the distributional assumptions made for γ_h and δ_h, the log likelihood function corresponding to the stochastic cost frontier can be expressed as follows:

$$L = \omega\text{-}H\ln\sigma_\xi + \sum_{h=1}^{H}\ln\Phi\left(\frac{\xi_h\lambda_\xi}{\sigma_\xi}\right) - \frac{1}{2\sigma_\xi^2}^2\sum_{h=1}^{H}\xi_h^2 \qquad (14)$$

where ω is a constant, $\sigma_\xi = \left(\sigma_\gamma^2 + \sigma_\delta^2\right)^{1/2}$ and $\lambda_\xi = \sigma_\delta / \sigma_\gamma$. Point estimates of each producer's overall inefficiency can be generated as follows:

$$OI_h = \left[1 - \frac{\mathbf{X}_h\mathbf{'B}}{\mathbf{X}_h\mathbf{'B} + \delta_h}\right] \Bigg/ \left[w_2 - W_{\mathbf{k}}(y, \mathbf{K}, \mathbf{L}, \frac{w_2}{w_1}, \frac{\mathbf{c}}{w_1}, t)\right] \qquad (15)$$

where $\mathbf{X}_h$ and $\mathbf{B}$ are the vectors of explanatory variables and parameter estimates respectively, $OE_h = \frac{\mathbf{X}_h\mathbf{'B}}{\mathbf{X}_h\mathbf{'B} + \delta_h}$ is a measure of overall efficiency, δ_h is replaced by its conditional expectation $E\left(\delta_h / \xi_h\right) = \sigma^{**}\left[\frac{\phi\left(\xi_h \lambda_\xi / \sigma_\xi\right)}{1 - \Phi\left(-\xi_h \lambda_\xi / \sigma_\xi\right)} - \frac{\xi_h \lambda_\xi}{\sigma_\xi}\right]$ and $\sigma_{**}^2 = \sigma_\delta^2 \sigma_\gamma^2 / \sigma_\xi^2$. Once the dynamic cost and technical efficiency measures are generated, one can estimate allocative efficiency through (6).

4. Empirical application

Our empirical application focuses on a sample of specialized dairy farms in Holland. Farm-level data are obtained from the European Commission's Farm Accountancy Data Network (FADN) and cover the period 1995-2005. To ensure that milk output is the main farm output, we select those farms whose milk sales represent at least 80% of total farm income. The dataset is an unbalanced panel that contains 2,614 observations on 639 farms that, on average, stay in the sample during 4 years.

In order to keep the vector of parameters to estimate to a manageable size, we distinguish one output, two variable inputs, two quasi-fixed inputs and two fixed inputs. Output (y) is defined as a farm's total output and includes livestock and livestock products, crops and crop products and other output. The two variable inputs are variable costs other than feed (x_1) and feed expenses (x_2). Variable x_1 is thus an aggregate input that includes veterinary expenses, energy, contract work, crop-specific costs and other variable input costs. Breeding livestock is considered as a quasi-fixed input (K_1). Machinery and buildings, also defined as quasi-fixed inputs, are aggregated into K_2. Variables y, x_1, x_2, K_1 and K_2 are measured at constant 1995 prices. Total utilized agricultural area (L_1), measured in hectares, and total labor input (L_2), which is mainly composed of family labor and measured in annual working units (AWU), are assumed to be fixed inputs.

Since output and input prices are unavailable from FADN, country-level price indices are taken from Eurostat's New Cronos Dataset. Netputs measured in monetary values are defined as implicit quantity indices by computing the ratio of value to its corresponding Tornqvist price index. Depreciation rates considered for buildings, machinery and breeding livestock are 3%, 10% and 25% respectively. The interest rate (r) is defined as the average, over the period 1995-2005, of the annual interest rate for 10 years' maturity government bonds (Eurostat) and is equal to 4.97%. Following previous research, we assume that the current price of a quasi-fixed input can be derived as the discounted sum of the future rents on the depreciated asset (Epstein and Denny, 1983; Pietola and Myers, 2000). Based on this assumption, the rental cost price of capital is defined as $c_i = (r + \delta_i) z_i$, where δ_i is the quasi-fixed asset depreciation rate and z_i is the quasi-fixed asset price (defined as a Tornqvist price index).

With $M = 1$ $C = 2$, $F = 2$ and $N = 2$ the parameter-restricted input distance function can be expressed as:

$$\vec{D}_h^i(y,\mathbf{K},\mathbf{L},\mathbf{x},\mathbf{I},t;\mathbf{1},\mathbf{1}) = a + a_y y + a_{L1}L_1 + a_{L2}L_2 + a_{x1}\left(I_1 + x_1\right) + a_{x2}\left(I_1 + x_2\right) + a_{I2}\left(-I_1 + I_2\right)$$
$$-I_1 + a_{k1}K_1 + a_{k2}K_2 + \frac{1}{2}a_{yy}y^2 + \frac{1}{2}a_{L1L1}L_1^2 + a_{L1L2}L_1L_2 + \frac{1}{2}a_{L2L2}L_2^2 +$$
$$+a_{x1x1}\left(\frac{I_1^2}{2} + \frac{x_1^2}{2} + I_1x_1\right) + a_{x2x2}\left(\frac{I_2^2}{2} + \frac{x_2^2}{2} + I_2x_2\right) + a_{x1x2}\left(\frac{I_1^2}{2} + \frac{I_2^2}{2} + x_1x_2 + I_1x_1 + I_2x_2\right) +$$
$$a_{I2x1}\left(-\frac{I_1^2}{2} + \frac{I_2^2}{2} - I_1x_1 + I_2x_1\right) + a_{I1x2}\left(\frac{I_1^2}{2} - \frac{I_2^2}{2} + I_1x_2 - I_2x_2\right) + a_{I1I2}\left(-\frac{I_1^2}{2} + I_1I_2 - \frac{I_2^2}{2}\right) + \quad (16)$$
$$\frac{1}{2}a_{K1K1}K_1^2 + a_{K1K2}K_1K_2 + \frac{1}{2}a_{K2K2}K_2^2 + a_{yI2}(-yI_1 + yI_2) + a_{yx1}(yI_1 + yx_1) + a_{yx2}(yI_1 + yx_2) +$$
$$a_{yK1}yK_1 + a_{yK2}yK_2 + a_{I1K2}(-I_1K_1 + I_1K_2) + a_{I2K1}(-I_1K_1 + I_2K_1) + a_{I2K2}(-I_1K_1 + I_2K_2) +$$
$$a_{K1x2}(I_1K_1 + K_1x_2) + a_{K2x1}(I_1K_1 + K_2x_1) + a_{K2x2}(I_1K_1 + K_2x_2) + a_{K1x1}(I_1K_1 + K_1x_1)$$
$$a_{yL1}yL_1 + a_{yL2}yL_2 + a_{L1I2}(-L_1I_1 + L_1I_2) + a_{L2I1}(-L_1I_1 + L_2I_1) + a_{L2I2}(-L_1I_1 + L_2I_2) +$$
$$a_{L1x1}(L_1I_1 + L_1x_1) + a_{L1x2}(L_1I_1 + L_1x_2) + a_{L2x1}(L_1I_1 + L_2x_1) + a_{L2x2}(L_1I_1 + L_2x_2) +$$
$$a_{L1K1}L_1K_1 + a_{L2K1}L_2K_1 + a_{L1K2}L_1K_2 + a_{L2K2}L_2K_2 + a_t t$$

and the cost frontier function to be estimated is:

$$C = r\Big\{b_0 + b_y y + b_{w2}\frac{w_2}{w_1} + b_{c1}\frac{c_1}{w_1} + b_{c2}\frac{c_2}{w_1} + b_{L1}L_1 + b_{L2}L_2 + \frac{1}{2}b_{yy}y^2 + \frac{1}{2}b_{w2w2}\left(\frac{w_2}{w_1}\right)^2 + \frac{1}{2}b_{c1c1}\left(\frac{c_1}{w_1}\right)^2 +$$
$$b_{c1c2}\frac{c_1}{w_1}\frac{c_2}{w_1} + \frac{1}{2}b_{c2c2}\left(\frac{c_2}{w_1}\right)^2 + \frac{1}{2}b_{L1L1}L_1^2 + b_{L1L2}L_1L_2 + \frac{1}{2}b_{L2L2}L_2^2 + b_{yw2}y\frac{w_2}{w_1} + b_{yc1}y\frac{c_1}{w_1} + b_{yc2}y\frac{c_2}{w_1} +$$
$$b_{yL1}yL_1 + b_{yL2}yL_2 + b_{w2c1}\frac{w_2}{w_1}\frac{c_1}{w_1} + b_{w2c2}\frac{w_2}{w_1}\frac{c_2}{w_1} + b_{w2L1}\frac{w_2}{w_1}L_1 + b_{w2L2}\frac{w_2}{w_1}L_2 + b_{c1L1}\frac{c_1}{w_1}L_1 + b_{c2L1}\frac{c_2}{w_1}L_1 +$$
$$b_{c1L2}\frac{c_1}{w_1}L_2 + b_{c2L2}\frac{c_2}{w_1}L_2\Big\} +$$
$$b_{k1}(rK_1 - \dot{K}_1) + b_{k2}(rK_2 - \dot{K}_2) + b_{k1k1}K_1(\frac{r}{2}K_1 - \dot{K}_1) +$$
$$b_{k1k2}\left\{K_1(\frac{r}{2}K_2 - \dot{K}_2) + K_2(\frac{r}{2}K_1 - \dot{K}_1)\right\} + b_{k2k2}K_2(\frac{r}{2}K_2 - \dot{K}_2) + b_{yk1}y(rK_1 - \dot{K}_1) +$$
$$b_{yk2}y(rK_2 - \dot{K}_2) + b_{w2k1}\frac{w_2}{w_1}(rK_1 - \dot{K}_1) + b_{w2k2}\frac{w_2}{w_1}(rK_2 - \dot{K}_2) + b_{k1c1}\frac{c_1}{w_1}(rK_1 - \dot{K}_1) +$$
$$b_{k2c1}\left\{\frac{c_1}{w_1}\left(rK_2 - \dot{K}_2\right) + \frac{c_2}{w_1}\left(rK_1 - \dot{K}_1\right)\right\} + b_{k2c2}\frac{c_2}{w_1}(rK_2 - \dot{K}_2) + b_{k1L1}L_1(rK_1 - \dot{K}_1) +$$
$$b_{k1L2}L_2(rK_1 - \dot{K}_1) + +b_{k2L1}L_1(rK_2 - \dot{K}_2) + b_{k2L2}L_2(rK_2 - \dot{K}_2) + b_t(rt - 1) + \xi_h \quad (17)$$

5. Results

Table 1 provides descriptive statistics for the variables used in the analysis. Farms' total output quantity index (y) has an average of almost 200 thousand per year. The mean quantity index representing total variable expenses (x_1 and x_2) is below 90 thousand, with feed expenses contributing 40% to this quantity. The observed long-run cost represents almost 70% of total output. The breeding livestock quantity index (K_1) is, on average, almost 69 thousand. While breeding livestock gross investments are substantial (I_1), net investments ($\dot{K}_1$) represent only 0.25% of K_1, which is due to the milk quota system regulating EU's dairy sector and limiting this sector's growth. While the milk quota places a strong cap on the growth of the dairy herd, it does not prevent modernization of dairy holdings that, on average, have net investments in machinery and buildings of almost 7% per year.

Table 2 provides parameter estimates of the directional distance function. Almost 70% of the parameters are statistically significant. As expected, the first derivatives of the directional distance function (table 3), suggest that the distance increases with an increase in variable, quasi-fixed and fixed inputs, while it decreases with an increase in output and investment demand. In other words, dynamic technical inefficiencies worsen when a farm requires more input to produce the same amount of output and gross investment, and improve when output and gross investment grow, keeping input use constant.

First derivatives are computed at the data means and Monte Carlo Bootstrapping techniques are used to generate their variances. We utilize 500 pseudo-samples of the same size as the original sample, drawn with replacement. We then estimate both the distance and the cost function and derive their first derivatives (calculated at constant values, i.e., at the means of the variables from the original sample). Replicated estimates of these derivatives are then used to derive their variance-covariance matrix.[2]

The Luenberger productivity change indicator (PC) (Chambers, 2002) is computed and decomposed into the efficiency changes (EC) and the technical efficiency change (TC) indicators. Results suggest a decline in productivity over the period of analysis (the PC has a mean value of -0.11), which can be attributed to a decline in the efficiency (EC = -0.21), not fully compensated by a positive technical change component (TC = 0.10). The progressive transformation of the Common Agricultural Policy (CAP) from a policy mainly based on price supports, to a policy based on (partially and fully) decoupled payments may explain a progressive reduction of the incentive of farmers to operate efficiently (Serra et al., 2008).

Estimation of the cost frontier model is presented in table 4. More than half of parameter estimates are statistically significant. Compatible with economic theory, the first derivatives of function $C_h(.)$ show that the cost increases with normalized variable ($\frac{w_2}{w_1}$) and quasi-fixed input prices ($\frac{c_1}{w_1}$ and $\frac{c_2}{w_1}$), while it decreases with the capital stock (table 5).

Dynamic technical, allocative and overall inefficiency estimates are presented in table 6. The average cost inefficiency (OI_h) over the period studied is 0.12, involving the possibility to produce the same amount of output with long-run cost savings on the order of 12%. Cost inefficiency is mainly due to technical inefficiency (TI_h^i) which is on the order of 0.11 and which suggests that there is scope for an 11% cost reduction through a more efficient use of inputs.

[2] It is noteworthy that our non-parametric bootstrap approach is robust to misspecification issues, including heteroskedasticity.

Our dynamic technical inefficiency scores are compatible though not directly comparable with static measures generated by previous research. Reinhard, Lovell and Thjissen (1999) assess, among other issues, technical efficiency of a sample of Dutch dairy farms through a production frontier and obtain average inefficiency values of almost 0.11. Using a shadow cost system approach, Reinhard and Thjissen (2000) derive, also for a sample of Dutch dairy farms, technical inefficiency scores on the order of 0.15. Kumbhakar et al. (2007) obtain inefficiency scores of 0.13 for a sample of Spanish dairy farms based on a nonparametric stochastic frontier. Sipiläinen and Oude Lansink (2005) use a stochastic frontier distance function and derive slightly higher inefficiency measures (0.17) for a sample of Finnish dairy farms.

Allocative inefficiency derived by our analysis (AI_h), with an average score of 0.01, shows little scope for cost reduction through an improved input mix given market prices. This indicates that Dutch dairy farmers are long-run cost minimizers. While we find allocative inefficiency to represent only around 9% of overall inefficiency, Silva and Oude Lansink (2009) find a deficient allocation of inputs relative to their market prices to generate 22% of overall inefficiency for a sample of Dutch glasshouse horticulture firms. Their allocative inefficiencies are on the order of 0.1. However, Reinhard and Thjissen (2000) find much lower allocative inefficiencies (below 0.5) for a sample of Dutch dairy farms.

6. Concluding remarks

The economics literature on efficiency measurement has traditionally ignored the adjustment of quasi-fixed inputs to their long-run equilibrium and time interdependence of production decisions. Recent proposals of dynamic efficiency measurement have been mainly developed in the framework of the nonparametric DEA, being the parametric approaches very scarce.

Up to date, Rungsuriyawiboon and Stefanou (2007) constitutes the only published structural parametric approach to dynamic efficiency measurement. Our analysis contributes to the literature by parametrically estimating the model proposed by Silva and Oude Lansink (2009), which generates dynamic efficiency measures based on a directional distance function and the duality between this function and the optimal value function. We propose an econometric estimation of the overall, technical and allocative efficiency measures proposed by these authors.

The empirical applicability of this proposal is illustrated by assessing dynamic efficiencies for a sample of Dutch dairy farms observed over the period 1995-2005. Dynamic efficiency ratings are compatible with static ratings derived by previous research. Average dynamic cost inefficiency indicates the possibility to accomplish long-run cost savings on the order of 12%. These cost savings are to be mainly achieved through a reduction in input use. An improved input mix given market prices offers, on the contrary, little scope for cost reduction.

References

Ahn, S.C. and Sickles, R.C. (2000). Estimation of long-run inefficiency levels: a dynamic frontier approach. *Econometric Reviews* 19: 461-492.

Chambers, R.G., Chung, Y. and Fare, R. (1998). Profit, directional distance functions, and nerlovian efficiency. *Journal of Optimization Theory and Applications* 98: 351-364.

Chambers, R.G. (2002). Exact nonradial input, output, and productivity measurement. *Economic Theory* 20: 751-765.

Epstein L. G., Denny M. (1983). The multivariate flexible accelerator model: its empirical restrictions and an application to U.S. manufacturing. *Econometrica* 51: 647–674.

Färe, R., Grosskopf, S., Noh, D-W, and Weber, W. (2005). Characteristics of a polluting technology: theory and practice. *Journal of Econometrics* 126: 469-492.

Kumbhakar, S. and Lovell, K. (2000). *Stochastic Frontier Analysis.* Cambridge: Cambridge University Press.

Kumbhakar, S., Park, B., Simar, L., Tsionas, E. (2007). Nonparametric stochastic frontiers: a local maximum likelihood approach. *Journal of Econometrics* 137: 1-27.

Nemoto, J., Goto, N. (1999). Dynamic data envelopment analysis: modelling intertemporal behavior of a firm in the presence of productive inefficiencies. *Economic Letters* 64:51-56.

Nemoto, J., Goto, N. (2003). Measurement of dynamic efficiency in production: an application of data envelopment analysis to Japanese electric utilities. *Journal of Productivity Analysis* 19: 191-210.

Pietola K., Myers R. J. (2000). Investment under uncertainty and dynamic adjustment in the Finnish pork industry. *American Journal of Agricultural Economics* 82: 956–967.

Reinhard, S., Lovell, K., Thijssen, G. (2000). Econometric estimation of technical and environmental efficiency: an application to Dutch dairy farms. *American Journal of Agricultural Economics* 27: 167-186.

Reinhard, S., Thijssen, G. (2000). Nitrogen efficiency of Dutch dairy farms: a shadow cost system approach. *European Review of Agricultural Economics* 27: 167-186.

Rungsuriyawiboon, S. and Stefanou, S. (2007). Dynamic efficiency estimation: an application to U.S. electric utilities. *Journal of Business & Economic Statistics* 25: 226-238.

Sengupta, J.K. (1995). *Dynamics of Data Envelopment Analysis*. Dordrecth: Kluwer Academic Publishers.

Serra, T., Zilberman, D. and Gil, J.M. (2008). Farm's technical inefficiencies in the presence of government programs. *The Australian Journal of Agricultural and Resource Economics* 52: 57-76.

Silva, E. and Oude Lansink, A. (2009). Dynamic efficiency measurement: a directional distance function approach. Unpublished manuscript. Wageningen: Wageningen University.

Silva, E. and Stefanou, S. (2003). Nonparametric dynamic production analysis and the theory of cost. *Journal of Productivity Analysis* 19: 5-32.

Silva, E. and Stefanou, S. (2007). Nonparametric dynamic efficiency measurement: theory and application. *American Journal of Agricultural Economics* 89: 398-419.

Sipiläinen, T. and Oude Lansink, A. (2005). Learning in organic farming – an application on Finnish dairy farms. Paper presented at the XIth Congress of the European Association of Agricultural Economists (EAAE), Copenhagen, Denmark, August 24-27, 2005.

Tsionas, M. (2006). Inference in dynamic stochastic frontier models. *Journal of Applied Econometrics* 21: 669-676.

Table 1. Descriptive statistics for the variables used in the analysis

Variable		Mean	Standard deviation
y	Total output (index)	199,665.76	115,708.47
C	Observed long-run cost (index)	137,006.94	75,100.78
K_1	Breeding livestock (index)	68,747.85	39,215.14
K_2	Buildings and machinery (index)	204,077.17	141,387.32
L_1	Land (hectares)	44.73	24.18
L_2	Labour (AWU)	1.71	0.64
x_1	Variable inputs, except feed (index)	52,075.09	28,278.93
x_2	Feed (index)	34,513.88	21,574.47
I_1	Gross investments in breeding livestock (index)	17,358.42	13,565.17
I_2	Gross investments in machinery and buildings (index)	24,754.31	53,066.53
$\dot{K}_1$	Net investments in breeding livestock (index)	171.46	7,115.17
$\dot{K}_2$	Net investments in machinery and buildings (index)	13,851.36	49,641.54
p	Output price (index)	0.99	0.04
w_1	Variable inputs' price (excluding feed) (index)	1.16	0.11
w_2	Feed price (index)	0.99	0.04
c_1	Breeding livestock rental price (index)	0.27	0.02
c_2	Machinery and buildings rental price (index)	0.12	0.01

Table 2. Directional distance function parameter estimates

Parameter	Estimate	Standard Error	Parameter	Estimate	Standard Error
a	-4.85E-02**	2.39E-02	a_{yK1}	2.28E-02	7.90E-02
a_{y}	-1.02E+00**	5.71E-02	a_{yK2}	-6.53E-03	6.60E-02
a_{L1}	3.70E-01**	4.80E-02	a_{I1K2}	-2.28E-02	2.45E-02
a_{L2}	-1.31E-01**	4.84E-02	a_{I2K1}	2.61E-02	3.02E-02
a_{x1}	3.87E-01**	3.33E-02	a_{I2K2}	-1.84E-02	3.03E-02
a_{x2}	5.49E-01**	3.67E-02	a_{K1x2}	-4.73E-02	7.43E-02
a_{I2}	-1.72E-02**	8.63E-03	a_{K2x1}	-8.60E-02*	4.51E-02
a_{k1}	-6.28E-02	6.90E-02	a_{K2x2}	2.08E-02	3.93E-02
a_{k2}	4.96E-02*	3.01E-02	a_{K1x1}	1.94E-01**	7.49E-02
a_{yy}	5.48E-01**	1.55E-01	a_{yL1}	1.40E-01*	7.97E-02
a_{L1L1}	1.01E-01*	5.48E-02	a_{yL2}	-1.99E-01**	7.65E-02
a_{L1L2}	-1.56E-01**	4.76E-02	a_{L1I2}	-2.63E-01**	2.35E-02
a_{L2L2}	-4.83E-02	5.77E-02	a_{L2I1}	-1.37E-01**	3.55E-02
a_{x1x1}	-3.50E-01**	4.45E-02	a_{L2I2}	2.65E-01**	2.37E-02
a_{x2x2}	-1.13E-01*	6.24E-02	a_{L1x1}	-2.94E-01**	4.78E-02
a_{x1x2}	2.23E-01**	3.92E-02	a_{L1x2}	-1.03E-01**	4.95E-02
a_{I2x1}	1.27E-02*	5.94E-03	a_{L2x1}	4.24E-01**	5.30E-02
a_{I1x2}	1.18E-01**	3.95E-02	a_{L2x2}	-2.92E-02	5.85E-02
a_{I1I2}	4.50E-03	4.79E-03	a_{L1K1}	-1.54E-01**	7.85E-02
a_{K1K1}	-2.36E-01*	1.25E-01	a_{L2K1}	2.60E-01**	8.17E-02
a_{K1K2}	1.69E-02	5.13E-02	a_{L1K2}	1.34E-02	3.53E-02
a_{K2K2}	-3.85E-03	2.25E-02	a_{L2K2}	1.93E-02	3.68E-02
a_{yI2}	-1.35E-02	1.19E-02	a_{t}	7.32E-02**	6.44E-03
a_{yx1}	1.03E-01*	6.19E-02	σ_{ε}	1.97E-01**	8.15E-03
a_{yx2}	-2.18E-01**	7.85E-02	λ_{ε}	1.53E+00**	2.12E-01

Note: *(**) denotes statistical significance at the 10(5%) level

Table 3. Properties of the directional distance function

Variable	Mean	Standard deviation
$\partial \vec{D}_h^i(.) / \partial y$	-7.41E-01	2.46E-02
$\partial \vec{D}_h^i(.) / \partial I_1$	-1.20E-01	1.19E-02
$\partial \vec{D}_h^i(.) / \partial I_2$	-1.24E-02	6.59E-03
$\partial \vec{D}_h^i(.) / \partial x_1$	4.75E-01	1.61E-02
$\partial \vec{D}_h^i(.) / \partial x_2$	3.92E-01	1.71E-02
$\partial \vec{D}_h^i(.) / \partial K_1$	9.20E-02	2.94E-02
$\partial \vec{D}_h^i(.) / \partial K_2$	6.50E-03	1.11E-02
$\partial \vec{D}_h^i(.) / \partial L_1$	5.20E-02	2.07E-02
$\partial \vec{D}_h^i(.) / \partial L_2$	1.34E-03	1.44E-02

Table 4. Cost function parameter estimates

Parameter	Estimate	Standard Error	Parameter	Estimate	Standard Error
b_0	2.76E+00	1.49E+01	b_{c2L1}	3.27E+00	3.39E+00
b_y	5.74E+00	4.70E+00	b_{c1L2}	-1.36E+00	2.36E+00
b_{w2}	-6.04E+01**	1.30E+01	b_{c2L2}	-1.35E+00	2.92E+00
b_{c1}	-3.91E+00	9.13E+00	b_{k1}	-1.76E-03	1.44E-03
b_{c2}	2.67E+01**	9.30E+00	b_{k2}	3.64E-03	1.98E-02
b_{L1}	3.40E+00	4.25E+00	b_{k1k1}	-7.83E-04**	3.85E-04
b_{L2}	-6.22E+00*	3.67E+00	b_{k1k2}	-3.80E-04	2.75E-04
b_{yy}	2.59E+00*	1.59E+00	b_{k2k2}	-7.77E-03**	1.58E-03
b_{w2w2}	1.30E+02**	3.03E+01	b_{yk1}	1.71E-03**	6.01E-04
b_{c1c1}	8.61E+01**	2.62E+01	b_{yk2}	1.25E-02**	4.37E-03
b_{c1c2}	2.53E+00	6.14E+00	b_{w2k1}	1.78E-03*	1.08E-03
b_{c2c2}	-2.88E+01**	8.59E+00	b_{w2k2}	2.09E-02	1.47E-02
b_{L1L1}	1.76E+00*	9.93E-01	b_{k1c1}	-8.99E-04	1.04E-03
b_{L1L2}	1.18E+00	8.64E-01	b_{k2c1}	5.62E-04	1.13E-03
b_{L2L2}	6.78E-01	1.04E+00	b_{k2c2}	-4.37E-02**	1.70E-02
b_{yw2}	2.19E+01**	3.84E+00	b_{k1L1}	-5.26E-04	3.40E-04
b_{yc1}	-7.15E+00**	3.60E+00	b_{k1L2}	3.07E-04	3.18E-04
b_{yc2}	-4.32E+00	4.35E+00	b_{k2L1}	-1.14E-03	2.96E-03
b_{yL1}	-3.25E+00**	1.18E+00	b_{k2L2}	9.58E-03**	4.40E-03
b_{yL2}	1.00E+00	9.76E-01	b_t	-8.23E-01	6.95E-01
b_{w2c1}	-8.00E+01**	2.44E+01	σ_ξ	1.93E-01**	6.81E-03
b_{w2c2}	3.47E+00	7.65E+00	λ_ξ	1.69E+00**	1.96E-01
b_{w2L1}	-1.27E+01**	3.75E+00			
b_{w2L2}	6.58E+00**	3.30E+00			
b_{c1L1}	6.73E+00**	3.36E+00			

Note: *(**) denotes statistical significance at the 10(5%) level

Table 5. Properties of the cost function

Variable	Mean	Standard deviation
$\partial C(.)/\partial K_1$	-1.43E-03	2.40E-02
$\partial C(.)/\partial K_2$	-4.19E-03	2.24E-02
$\partial C(.)/\partial w_2$	4.37E-01	1.25E-01
$\partial C(.)/\partial c_1$	1.61E-01	5.49E-02
$\partial C(.)/\partial c_2$	1.16E-01	4.09E-02

Table 6. Efficiency ratings

Variable	Mean	Standard deviation
TE_h^i	0.892	0.105
OE_h	0.882	0.091
TI_h^i	0.107	0.105
OI_h	0.117	0.094
AI_h	0.010	0.097