Technical Appendix of "Peak-load Pricing with Different Types of Dispatchability"

Klaus Eisenack[*]*and Mathias Mier*[†]

November 7, 2019

Abstract

This technical appendix of Eisenack and Mier (2019) presents the proofs and calculations (Section 1), and the extension to perfectly correlated generation units (Section 2). See also Eisenack and Mier (2018).

Contents

[*]Resource Economics Group, Humboldt-Universität zu Berlin, Unter den Linden 6, 10099 Berlin, Germany, klaus.eisenack@hu-berlin.de, ph +49 30 2093-46360.

[†]Corresponding author. ifo Institute for Economic Research at the University of Munich, Center for Energy, Climate and Resources, Poschingerstr. 5, 81679 Munich, Germany, mier@ifo.de, ph +49 89 9224-1365.

1 Calculations and Proofs

1.1 Deriving the First-Order Conditions

One can generally solve the program by using Kuhn-Tucker conditions. Yet, as several of steps are common, we keep them brief. Start with Stage 4. If follows from the definition of excess demand that $x_0 = D - \tilde{x}_N - x_P - x_H > 0$ only if $\tilde{x}_N < D - x_P - x_H$. As $D, x_P, \tilde{x}_N$ are given in Stage 3, we can leave out expectations and the problem is to maximize w.r.t. x_H. We obtain the derivative

$$\frac{\partial J}{\partial x_H} = \begin{cases} -c_H + c_0 & > 0 \text{ for } \tilde{x}_N < D - x_P - x_H, \\ -c_H & < 0 \text{ else.} \end{cases} \tag{1}$$

The signs of the derivatives follow from the cost assumptions. Thus, highly dispatchable technologies are only employed if there would be excess demand otherwise. The optimal output is

$$x_H = \begin{cases} k_H & \text{for } \tilde{x}_N \in \Omega_4, \\ D - \tilde{x}_N - x_P & \text{for } \tilde{x}_N \in \Omega_3, \\ 0 & \text{else.} \end{cases} \tag{2}$$

In Stage 3, non-dispatchable production realizes, independently of all decision variables except k_N, so that

$$E[\tilde{x}_N] = ak_N \tag{3}$$

By using the definition of excess demand, (2), and conditional expectations, we obtain the expected outcome of Stage 3 as given by

$$E[x_H] = k_H \Pr_4 + E[D - x_P - \tilde{x}_N | \Omega_3] \Pr_3, \tag{4}$$

$$E[x_0] = E[D - x_P - \tilde{x}_N - x_H | \Omega_3] \Pr_4. \tag{5}$$

Now turn to Stage 2. Inserting expected output of all capacities and $E[x_0]$ into the maximand (Equation 1 in Eisenack and Mier (2019)) yields

$$\begin{aligned} E[J] = {} & U(D) - \sum_j b_j k_j - c_P x_P - c_N a k_N \\ & - c_H \left(k_H \Pr_4 + E[D - x_P - \tilde{x}_N | \Omega_3] \Pr_3 \right) \\ & - c_0 E[D - x_P - \tilde{x}_N - k_H | \Omega_4] \Pr_4. \end{aligned} \tag{6}$$

Due to the Inada conditions for $U(D)$, the first-order condition $\frac{\partial E[J]}{\partial D} = 0$ yields optimal marginal utility of

$$-\frac{\partial E[J]}{\partial D} = c_H \Pr{}_3 + c_0 \Pr{}_4 - U'(D). \tag{7}$$

By using that $\tilde{x}_N$ is boundedly integrable, the derivatives can be simplified by interchanging differentiation and expectation (see also Chao 1983, p. 182; Chao, 2011, p. 3952). We will use this feature repeatedly in the following. Here, it yields

$$\frac{\partial E[J]}{\partial x_P} = c_H \Pr{}_3 + c_0 \Pr{}_4 - c_P. \tag{8}$$

If the derivative is positive (negative), it is (not) beneficial to increase production of partially dispatchable technologies up to its capacity. Thus, depending on the sign of the derivative, either $x_P < k_P$ or $x_P = k_P$ in the optimum. Suppose that $x_P < k_P$ is optimal. Then $\frac{\partial E[J]}{\partial k_P} = -b_P < 0$. employing partially dispatchable technologies would never be beneficial so that $x_P = k_P = 0$, a contradiction to $x_P < k_P$. Consequently, $x_P = k_P$. The remaining first-order conditions are

$$\frac{\partial E[J]}{\partial k_N} = ac_H \Pr{}_3 + ac_0 \Pr{}_4 - b_N - ac_N, \tag{9}$$

$$\frac{\partial E[J]}{\partial k_P} = c_H \Pr{}_3 + c_0 \Pr{}_4 - b_P - c_P, \tag{10}$$

$$\frac{\partial E[J]}{\partial k_H} = -c_H \Pr{}_4 + c_0 \Pr{}_4 - b_H. \tag{11}$$

1.2 Proof of Proposition 1

Case without non-dispatchable technologies. We denote results for the case $k_P > k_N = 0$ with the superscript P. Demand is always met, $x_P^P + x_H^P = D^P$, because scheduled demand is fixed and excess demand is more costly than output from either partially or highly dispatchable technologies, $b_P + c_P, b_H + c_H < c_0$. This yields $x_0 = 0$ and $\Pr_4 = 0$. As the LRMC of partially dispatchable technologies are lower than the LRMC of highly dispatchable technologies, $b_P + c_P < b_H + c_H$, there is no highly dispatchable capacity in the optimum, i.e., $k_H = 0$ and $\Pr_3 = 0$. As excess capacity has no benefits, partially dispatchable technologies must produce at full capacity, $x_P^P = k_P^P = D^P$. The maximand is simplified to $J^P := U\left(D^P\right) - (b_P + c_P) D^P$. Thus, optimally scheduled demand D^P is characterized by the marginal condition $U'\left(D^P\right) = b_P + c_P$.

Case without partially dispatchable technologies. We denote results for the case $k_N > k_P = 0$ with superscript N. Solving (11) for Pr_4^N, using (11) in (9), and solving for Pr_{34}^N, we obtain

$$\mathrm{Pr}_4^N = \frac{b_H}{c_0 - c_H}, \tag{12}$$

$$\mathrm{Pr}_{34}^N = \frac{\frac{b_N}{a} + c_N - b_H}{c_H}. \tag{13}$$

It follows from $b_H + c_H < c_0$ and $b_H > 0$ that $\mathrm{Pr}_4 \in (0,1)$. Using (12) and (13) in the first-order condition (7) yields D^N, characterized by marginal utility $U'\left(D^N\right) = b_H + c_H \mathrm{Pr}_{34} = \frac{b_N}{a} + c_N$. We can further exploit $k_P = 0$ and (3) and (11) in the maximand (6) to obtain $J^N := U\left(D^N\right) - \left(\frac{b_N}{a} + c_N\right) D^N - \gamma k_N$ with

$$\gamma := (a - a_{34})\left(\frac{b_N}{a} + c_N\right) + (a_{34} - a_4)\, b_H, \tag{14}$$

where $\gamma > 0$ without loss of generality due to $a_4 < a_{34} < a$.

Comparison of cases. By denoting $\Delta U := U\left(D^N\right) - U\left(D^P\right)$ and $\Delta D := D^N - D^P$, it can be verified that $J^N = J^P$ if

$$\Delta C = \Psi := \frac{\left(\frac{b_N}{a} + c_N\right)\Delta D + \gamma k_N - \Delta U}{D^P}. \tag{15}$$

Positivity of Ψ. Suppose that $\Delta C = \Psi \leq 0$. Note that utility is strictly increasing, strictly concave, and fulfills Inada conditions, i.e., $U\left(D^N\right) - U'\left(D^N\right) D^N \leq U\left(D^P\right) - U'\left(D^P\right) D^P$. Additionally accounting for $\gamma > 0$ leads to $J^N < J^P$, a contradiction. We conclude that $\Psi > 0$.

1.3 Proof of Lemma 1

We only need to consider the situation with $k_P = 0$. Then, $\mathrm{Pr}_4 = \int_0^{D-k_H} f\left(\tilde{x}_N; k_N\right) d\tilde{x}_N$, $\mathrm{Pr}_3 = \int_{D-k_H}^{D} f\left(\tilde{x}_N; k_N\right) d\tilde{x}_N$, and $\mathrm{Pr}_{34} = \int_0^D f\left(\tilde{x}_N; k_N\right) d\tilde{x}_N$. By the Leibniz rule, we obtain the signs of the derivatives of $\mathrm{Pr}_3, \mathrm{Pr}_{34}, \mathrm{Pr}_4$ w.r.t. k_H, D as given in Table 1 below, as f is independent from k_H, D. The derivatives $\frac{\partial \mathrm{Pr}_{34}}{\partial k_N} = \int_0^D \frac{\partial f(\tilde{x}_N; k_N)}{\partial k_N} d\tilde{x}_N$ and $\frac{\partial \mathrm{Pr}_4}{\partial k_N} = \int_0^{D-k_H} \frac{\partial f(\tilde{x}_N; k_N)}{\partial k_N} d\tilde{x}_N$ are negative as they are determined over intervals bounded below by zero, because $\tilde{x}_N \geq 0$ by definition, and almost sure $\forall z: \omega(z) > 0$. The sign of $\frac{\partial \mathrm{Pr}_3}{\partial k_N} = \int_{D-k_H}^{D} \frac{\partial f(\tilde{x}_N; k_N)}{\partial k_N} d\tilde{x}_N$ is generally ambiguous because $\frac{\partial f(\tilde{x}_N; k_N)}{\partial k_N}$ can be higher or lower at D or $D - k_H$. Thus, every component of the Table in Lemma 1 has been shown.

	$\partial/\partial k_N$	$\partial/\partial k_H$	$\partial/\partial D$
$\Pr_{34}$	$\int_0^D \frac{\partial f(\tilde{x}_N;k_N)}{\partial k_N} d\tilde{x}_N < 0$	0	$f(D;k_N) > 0$
$\Pr_4$	$\int_0^{D-k_H} \frac{\partial f(\tilde{x}_N;k_N)}{\partial k_N} d\tilde{x}_N < 0$	$-f(D-k_H;k_N) < 0$	$f(D-k_H;k_N) > 0$
$\Pr_3$	not possible to show	$f(D-k_H;k_N) > 0$	$f(D;k_N) - f(D-k_H;k_N)$

Tab. 1: Partial derivatives of probabilities w.r.t. k_N, k_H, D

1.4 Proof of Proposition 2

The comparative statics can be derived from first-order conditions (7), (9), and (11). The total differential of these three conditions with respect to the dependent variables (k_N, k_H, D) and one parameter of interest (here: one of b_N, c_N, b_H, c_H, c_0) always yields an equation system. This needs to be solved to obtain the comparative statics of k_N, k_H, D with respect to the parameter. These solutions are eased by noting that the first-order conditions can be equivalently written as

$$F_H := (c_0 - c_H)\Pr_4 - b_H = 0, \tag{16}$$

$$F_D := \frac{b_N}{a} + c_N - U' = 0, \tag{17}$$

$$F_N := b_H + c_H \Pr_{34} - \frac{b_N}{a} - c_N = 0, \tag{18}$$

by considering the following: (16) just rewrites (11). (17) is obtained by solving (16) for $c_0 \Pr_4$, substituting into (9), and using that $\Pr_3 + \Pr_4 = \Pr_{34}$. (18) is obtained from (9), substituting $c_0 \Pr_4$, and simplifying as before. The partial derivatives are summarized in Table 2, where $\frac{\partial F_N}{\partial k_H} = 0$ is implied by Table 1 from Section 1.3.

	$\partial/\partial k_N$	$\partial/\partial k_H$	$\partial/\partial D$
F_N	$c_H \frac{\partial \Pr_{34}}{\partial k_N}$	0	$c_H \frac{\partial \Pr_{34}}{\partial D}$
F_H	$(c_0 - c_H) \frac{\partial \Pr_4}{\partial k_N}$	$(c_0 - c_H) \frac{\partial \Pr_4}{\partial k_H}$	$(c_0 - c_H) \frac{\partial \Pr_4}{\partial D}$
F_D	0	0	$-U''$

	$\partial/\partial b_N$	$\partial/\partial c_N$	$\partial/\partial b_H$	$\partial/\partial c_H$	$\partial/\partial c_0$
F_N	$-1/a$	-1	1	$\Pr_{34}$	0
F_H	0	0	-1	$-\Pr_4$	$\Pr_4$
F_D	$1/a$	1	0	0	0

Tab. 2: Partial derivatives of F_N, F_H, F_D.

This structure make the comparative statics for D easy to determine. Using implicit differentiation, we obtain $\frac{dD}{db_H} = -\frac{\partial F_D}{\partial b_H} / \frac{\partial F_D}{\partial D} = 0$, $\frac{dD}{dc_H} = \frac{dD}{dc_0} = 0$, $\frac{dD}{db_N} = \frac{1}{aU''} < 0$, and $\frac{dD}{dc_N} = \frac{1}{U''} < 0$. The total differential of (18) becomes, for any parameter, a straightforward equation because $\frac{\partial F_N}{\partial k_H} = 0$.

For b_N, e.g.,

$$0 = c_H \frac{\partial \Pr_{34}}{\partial k_N} dk_N + c_H \frac{\partial \Pr_{34}}{\partial k_H} dk_H + c_H \frac{\partial \Pr_{34}}{\partial D} dD - \frac{1}{a} db_N, \tag{19}$$

where $\frac{\partial \Pr_{34}}{\partial k_H} = 0$ as shown in Table 1. Solving for $\frac{dk_N}{db_N}$ by using $\frac{\partial \Pr_{34}}{\partial k_N}, \frac{\partial \Pr_{34}}{\partial D}$ from Table 1 and $\frac{dD}{db_N} = \frac{1}{aU''}$ leads to

$$\frac{dk_N}{db_N} = \frac{1}{a} \frac{U'' - c_H \frac{\partial \Pr_{34}}{\partial D}}{U'' c_H \frac{\partial \Pr_{34}}{\partial k_N}} > 0. \tag{20}$$

For the other parameters, the same steps yield $\frac{dk_N}{dc_N} = a \frac{dk_N}{db_N} > 0$, $\frac{dk_N}{db_H} = \left(-c_H \frac{\partial \Pr_{34}}{\partial k_N} \right)^{-1} > 0$, $\frac{dk_N}{dc_H} = \Pr_{34} \left(-c_H \frac{\partial \Pr_{34}}{\partial k_N} \right)^{-1} > 0$, and $\frac{dk_N}{dc_0} = 0$. For the comparative statics for k_H, we start with b_H, so that the total differential of (16) is

$$dF_H = (c_0 - c_H) \frac{\partial \Pr_4}{\partial k_N} dk_N + (c_0 - c_H) \frac{\partial \Pr_4}{\partial k_H} dk_H + (c_0 - c_H) \frac{\partial \Pr_4}{\partial D} dD - db_H = 0, \tag{21}$$

so that

$$\frac{\partial \Pr_4}{\partial k_N} \frac{dk_N}{db_H} + \frac{\partial \Pr_4}{\partial k_H} \frac{dk_H}{db_H} = \frac{1}{c_0 - c_H}, \tag{22}$$

where we have used that $\frac{dD}{db_H} = 0$. Now, the result for $\frac{dk_N}{db_H}$ and Table 1 can be used to determine

$$\frac{dk_H}{db_H} = \frac{c_H \frac{\partial \Pr_{34}}{\partial k_N} + (c_0 - c_H) \frac{\partial \Pr_4}{\partial k_N}}{c_H \frac{\partial \Pr_{34}}{\partial k_N} (c_0 - c_H) \frac{\partial \Pr_4}{\partial k_H}} < 0. \tag{23}$$

In the same way, we obtain $\frac{\partial k_H}{\partial c_H} = \Pr_{34} \frac{\partial k_H}{\partial b_H} < 0$, $\frac{\partial k_H}{\partial c_0} = -\Pr_4 \left((c_0 - c_H) \frac{\partial \Pr_4}{\partial k_H} \right)^{-1} > 0$, and

$$\frac{dk_H}{dc_N} = \frac{-U'' \frac{\partial \Pr_4}{\partial k_N} + c_H \left(\frac{\partial \Pr_4}{\partial k_N} \frac{\partial \Pr_{34}}{\partial D} - \frac{\partial \Pr_{34}}{\partial k_N} \frac{\partial \Pr_4}{\partial D} \right)}{U'' c_H \frac{\partial \Pr_{34}}{\partial k_N} \frac{\partial \Pr_4}{\partial k_H}} = a \frac{\partial k_H}{\partial b_N}, \tag{24}$$

where (24) has an ambiguous sign. The denominator is always negative, so that the numerator determines the sign. It follows that $\frac{dk_H}{db_N}, \frac{dk_H}{dc_N} < 0$ if and only if

$$\begin{aligned}
0 &< -U''\frac{\partial \Pr_4}{\partial k_N} + c_H\left(\frac{\partial \Pr_4}{\partial k_N}\frac{\partial \Pr_{34}}{\partial D} - \frac{\partial \Pr_{34}}{\partial k_N}\frac{\partial \Pr_4}{\partial D}\right) \\
-U'' &< -c_H\frac{\frac{\partial \Pr_4}{\partial k_N}\frac{\partial \Pr_{34}}{\partial D} - \frac{\partial \Pr_{34}}{\partial k_N}\frac{\partial \Pr_4}{\partial D}}{\frac{\partial \Pr_4}{\partial k_N}} \\
&= c_H\left(\frac{\partial \Pr_{34}}{\partial k_N}/\frac{\partial \Pr_4}{\partial k_N}\cdot\frac{\partial \Pr_4}{\partial D} - \frac{\partial \Pr_{34}}{\partial D}\right) \\
&= c_H\left(\frac{\partial \Pr_{34}}{\partial k_N}/\frac{\partial \Pr_4}{\partial k_N}\cdot f\left(D-k_H;k_N\right) - f\left(D;k_N\right)\right). \qquad (25)
\end{aligned}$$

1.5 Comparison to Standard Model

The first-order conditions are

$$-\frac{\partial E[J]}{\partial D} = c_0\Pr_4 + c_H\Pr_3 + c_P\Pr_2 - U'(D), \qquad (26)$$

$$\frac{\partial E[J]}{\partial k_H} = c_0\Pr_4 - c_H\Pr_4 - b_H, \qquad (27)$$

$$\frac{\partial E[J]}{\partial k_P} = c_0\Pr_4 + c_H\Pr_3 - c_P\Pr_{34} - b_P, \qquad (28)$$

$$\frac{\partial E[J]}{\partial k_N} = ac_0\Pr_4 + ac_H\Pr_3 + ac_P\Pr_2 - ac_N - b_N. \qquad (29)$$

Setting the first order conditions to zero and solving the system yields $U'(D) = b_P + c_P\Pr_{234}$ with

$$\Pr_4 = \frac{b_H}{c_0 - c_H}, \qquad (30)$$

$$\Pr_{34} = \frac{b_P - b_H}{c_H - c_P}, \qquad (31)$$

$$\Pr_{234} = \frac{\frac{b_N}{a} + c_N - b_P}{c_P}. \qquad (32)$$

1.6 Proof of Proposition 4

The proof starts from the assumption that $k_P \cdot k_N > 0$, and shows that this implies a specific relation for b_P, c_P, b_N, c_N. Thus, this relation is a necessary condition for both capacities being strictly positive. The objective is to maximize $E\left[J^{mul}\right] := \sum_t E[J_t] - \sum_j b_j k_j$, where $E[J_t] = U_t(D_t) - \sum_j c_j E\left[x_{jt}\right] - c_0 E[x_{0t}]$, and excess demand at time t is $x_{0t} = \max\left\{D_t - \sum_j x_{jt}, 0\right\}$. For convenience, we define $U_t'(D_t) := \frac{\partial U_t(D_t)}{\partial D_t}$. In Stages 2 to 4, this additive separable structure allows to maximize $E[J_t]$ separately for each period. Since $k_N > 0$ by assumption, we can rewrite the one-period results from (4) and (5) with time index, and obtain the derivatives

$$\begin{aligned}
E\left[x_{0t}\right] &= E\left[D_t - x_{Pt} - \tilde{x}_{Nt} - x_{Ht} | \Omega_{4t}\right] \Pr{}_{4t}, &(33)\\
E\left[x_{Ht}\right] &= k_H \Pr{}_{4t} + E\left[D_t - x_{Pt} - \tilde{x}_{Nt} | \Omega_{3t}\right] \Pr{}_{3t}, &(34)\\
-\frac{\partial E\left[J_t\right]}{\partial D_t} &= c_H \Pr{}_{3t} + c_0 \Pr{}_{4t} - U_t'\left(D_t\right), &(35)\\
\frac{\partial E\left[J_t\right]}{\partial x_{Pt}} &= c_H \Pr{}_{3t} + c_0 \Pr{}_{4t} - c_P. &(36)
\end{aligned}$$

Setting (35) to zero implies that $\forall t : U_t' = c_H \Pr_{3t} + c_0 \Pr_{4t}$. For an internal optimum, (36) is equal to zero as well, while for a corner solution $x_{Pt} = k_P$. Denote the subset of all periods with a corner solution by L, and $|L|$ is the number of periods in L. In Stage 1, the first-order condition for k_P then simplifies to

$$\frac{\partial E\left[J^{mul}\right]}{\partial k_P} = c_H \sum_{t \in L} \Pr_{3t} + c_0 \sum_{t \in L} \Pr_{4t} - b_P - |L| c_P = 0. \tag{37}$$

This condition is satisfied for at least one $k_P > 0$ as we assumed a positive partially dispatchable capacity to be optimal. Moreover, (35) being zero in any period implies $\sum_{t \in L} U_t'\left(D_t\right) = c_H \sum_{t \in L} \Pr_{3t} + c_0 \sum_{t \in L} \Pr_{4t}$. We thus obtain $\sum_{t \in L} U_t'\left(D_t\right) = b_P + |L| c_P$. The left-hand side of this equation depends on the values D_t. Since the optimal D_t depend on the cost parameters in turn, this is actually an equation that expresses a specific relation of the costs parameters. Both technology types are only employed in this boundary case. This shows that positive capacities of all technology types can only be optimal if cost parameters are in a specific relation to the optimal demand profile.

1.7 Proof of Proposition 5

Whereas $\tilde{x}_N \leq k_N$ is the available production of non-dispatchable technologies, $x_N \leq \tilde{x}_N$ denotes the actual output. Stage 4 is extended to $\max_{x_N, x_H} J$ s.t. $x_N \leq \tilde{x}_N, x_H \leq k_H$. The optimum depends on the random event: If $\tilde{x}_N \in \Omega_1$ or $\tilde{x}_N \in \Omega_2$, then $x_N = D - x_P, x_H = x_0 = 0$; if $\tilde{x}_N \in \Omega_3$, then $x_N = \tilde{x}_N, x_H = D - x_P - \tilde{x}_N, x_0 = 0$; if $\tilde{x}_N \in \Omega_4$, then $x_N = \tilde{x}_N, x_H = k_H, x_0 = D - x_P - \tilde{x}_N - k_H$.

Thus,

$$
\begin{aligned}
E\left[x_0\right] &= E\left[D - x_P - \tilde{x}_N - k_H \middle| \Omega_4\right] \Pr{}_4, && (38)\\
E\left[x_H\right] &= E\left[D - x_P - \tilde{x}_N \middle| \Omega_3\right] \Pr{}_3 + k_H \Pr{}_4, && (39)\\
E\left[x_N\right] &= E\left[\tilde{x}_N \middle| \Omega_3\right] \Pr{}_3 + E\left[\tilde{x}_N \middle| \Omega_4\right] \Pr{}_4 + \left(D - x_P\right) \Pr{}_{12}, && (40)\\
E\left[J\right] &= U\left(D\right) - \sum_j b_j k_j - c_P x_P \\
&\quad - c_N\left(E\left[\tilde{x}_N \middle| \Omega_3\right] \Pr{}_3 + E\left[\tilde{x}_N \middle| \Omega_4\right] \Pr{}_4 + \left(D - x_P\right) \Pr{}_{12}\right) \\
&\quad - c_H\left(k_H \Pr{}_4 + E\left[D - \tilde{x}_N - x_P \middle| \Omega_3\right] \Pr{}_3\right) \\
&\quad - c_0 E\left[D - \tilde{x}_N - x_P - k_H \middle| \Omega_4\right) \Pr{}_4. && (41)
\end{aligned}
$$

The derivatives for the remaining decision variables are:

$$
\begin{aligned}
-\frac{\partial E\left[J\right]}{\partial D} &= c_N \Pr{}_{12} + c_H \Pr{}_3 + c_0 \Pr{}_4 - U'\left(D\right), && (42)\\
\frac{\partial E\left[J\right]}{\partial x_P} &= c_N \Pr{}_{12} + c_H \Pr{}_3 + c_0 \Pr{}_4 - c_P, && (43)\\
\frac{\partial E\left[J\right]}{\partial k_H} &= -b_H - c_H \Pr{}_4 + c_0 \Pr{}_4, && (44)\\
\frac{\partial E\left[J\right]}{\partial k_P} &= c_N \Pr{}_{12} + c_H \Pr{}_3 + c_0 \Pr{}_4 - b_P - c_P, && (45)\\
\frac{\partial E\left[J\right]}{\partial k_N} &= a\left(c_H - c_N\right) \Pr{}_3 + a\left(c_0 - c_N\right) \Pr{}_4 - b_N. && (46)
\end{aligned}
$$

Setting the last two expressions to zero and using $\Pr_{12} = 1 - \Pr_3 - \Pr_4$, $\Pr_3 + \Pr_4 = \Pr_{34}$ yields an overdetermined equation system (two equations for $\Pr_{34}$). This can only be solved for a boundary case with specific cost parameters.

1.8 Proof of Proposition 6

The analysis of the case of perfect correlation is equivalent until the first-order conditions need to get determined in Stage 1. Condition (9) changes to

$$
\frac{\partial E\left[J\right]}{\partial k_N} = a_3 c_H \Pr{}_3 + a_4 c_0 \Pr{}_4 - b_N - a c_N. \qquad (47)
$$

The derivatives (47) and (10) cannot become zero at the same time. The only exception is the boundary case, where the difference between partially and non-dispatchable LRMC becomes exactly $\Delta C = \Phi$, with

$$\Phi := \frac{a - a_3}{a_3}\left(\frac{b_N}{a} + c_N\right) + \frac{a_3 - a_4}{a_3} c_0 \,\mathrm{Pr}_4. \tag{48}$$

2 The Case of Perfectly Correlated Random Generation Units

The model in Eisenack and Mier (2019) focuses on an important special case for the random variable: marginal generating units are stochastically independent. Another (polar) case is to assume that marginal generating units are perfectly correlated, that is, each unit produces the same amount of output at a given point in time. This case of supply uncertainty has not been considered in the peak-load pricing literature to our knowledge so far. This technical appendix presents the results for the correlated case, and describes similarities and differences to the case of stochastically independent generation units. We are optimistic that if some results hold for both extremes, they might be robust to more general conditions.

2.1 Production and Capacity Decisions

As in the case of stochastically independent correlated generation units (*case of independence*), we define $\tilde{x}_N = \int_0^{k_N} \omega(z)\, dz$, where $\omega(z)$ are stochastically identically distributed random variables with realizations $\omega(z) \in [0,1]$ and availability factor $a = E[\omega(z)]$. In contrast to the case of independence, we now assume that production of marginal generating units is perfectly correlated, i.e., that each unit z produces the same output at a given point in time (*case of perfect correlation*). It will be convenient to denote *average conditional production* by $a_c := \frac{E[\tilde{x}_N|\Omega_c]}{k_N}$.

The analysis of the case of perfect correlation is equivalent to the case of independence until the first-order conditions need to get determined in Stage 1. We obtain

$$
\begin{aligned}
-\frac{\partial E[J]}{\partial D} &= c_H \Pr{}_3 + c_0 \Pr{}_4 - U'(D) && (49)\\
\frac{\partial E[J]}{\partial k_N} &= a_3 c_H \Pr{}_3 + a_4 c_0 \Pr{}_4 - b_N - a c_N, && (50)\\
\frac{\partial E[J]}{\partial k_P} &= c_H \Pr{}_3 + c_0 \Pr{}_4 - b_P - c_P, && (51)\\
\frac{\partial E[J]}{\partial k_H} &= -c_H \Pr{}_4 + c_0 \Pr{}_4 - b_H. && (52)
\end{aligned}
$$

Again, the first-order conditions (50) and (51) cannot become zero at the same time. The only exception is the boundary case, where the difference between partially and non-dispatchable long-run marginal costs (LRMC) becomes exactly $\Delta C = \Phi$, with

$$\Phi := \frac{a - a_3}{a_3}\left(\frac{b_N}{a} + c_N\right) + \frac{a_3 - a_4}{a_3} c_0 \Pr{}_4. \tag{53}$$

The next step is to determine whether a capacity decision with $k_P > k_N = 0$ or with $k_N > k_P = 0$ is optimal. Start with $k_P > k_N = 0$ and denote results for this case with the superscript P. We know that $x_P^P + x_H^P = D^P$. This yields $x_0 = 0$ and $\Pr_4 = 0$. From $b_P + c_P < b_H + c_H$, we obtain $k_H = 0$ and $\Pr_3 = 0$. As excess capacity has no benefits but costs, we have $x_P^P = k_P^P = D^P$ and $J^P := U\left(D^P\right) - \left(b_P + c_P\right) D^P$, where D^P is characterized by $U'\left(D^P\right) = b_P + c_P$.

Next consider $k_N > k_P = 0$ and denote results with the superscript N. Solving the first-order conditions (52) and (50) for $\Pr_4$ and $\Pr_3$, respectively, yields

$$\Pr_4 = \frac{b_H}{c_0 - c_H}, \tag{54}$$

$$\Pr_3 = \frac{b_N + ac_N - a_4 c_0 \Pr_4}{a_3 c_H}. \tag{55}$$

Using (54) and (55) in (49), yields D^N, characterized by $U'\left(D^N\right) = \frac{b_N}{a} + c_N + \Phi$. We obtain $J^N := U\left(D^N\right) - U'\left(D^N\right) D^N$. Demand is optimally scheduled so that marginal utility is equal to the LRMC of non-dispatchable technologies plus a mark-up Φ. We thus call Φ the *correlation mark-up*. Its sign is generally inconclusive. The correlation mark-up reflects the effect of raising capacity of non-disptachables on the different events. If an additional marginal generating unit is employed in the case of independence, then $\frac{dE[\tilde{x}_N|\Omega_c]}{dk_N} = a$ for all interval of events Ω_c. In the case of perfect correlation, however, it also depends on scheduled demand how expected conditional output changes.

By denoting $\Delta U := U\left(D^N\right) - U\left(D^P\right)$ and $\Delta D := D^N - D^P$, it can be verified that $J^N = J^P$ if

$$\Delta C = \Psi := \Phi + \frac{U'\left(D^N\right)\Delta D - \Delta U}{D^P}. \tag{56}$$

In addition, observe the following. Consider the boundary case with $\Delta C = \Phi$. It follows that $U'\left(D^N\right) = U'\left(D^P\right)$ and, thus, $\Delta D, \Delta U = 0$, because the differences in scheduled demand and utility vanish if the marginal utility without any partially dispatchable capacity is the same as without any non-dispatchable capacity. We obtain $\Psi = \Phi$. This yields an analogue to Proposition 1 in Eisenack and Mier (2019):

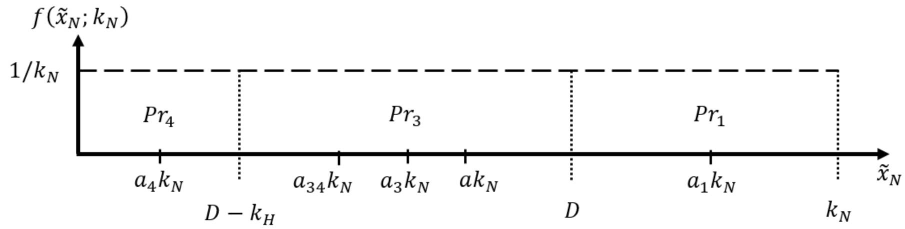
Proposition. *In the case of perfect correlation, the following cases can be distinguished:*

(i) If $\Delta C < \Phi$, then $x_P = k_P = D$ and $k_N = k_H = 0$ with $U'(D) = b_P + c_P$ and $\Pr_3 = \Pr_{34} = 0$.

(ii) If $\Delta C > \Phi$, then $k_P = 0$ and $k_N, k_H > 0$ with $U'(D) = \frac{b_N}{a} + c_N + \Phi$ and $\Pr_4 = \frac{b_H}{c_0 - c_H} \in (0,1)$, $\Pr_{34} = \frac{\frac{b_N}{a} + c_N - b_H}{c_H}$.

2.2 Special Case of Uniform Distribution

The case of perfect correlation can be further illustrated if we additionally assume that output $\tilde{x}_N$ is uniformly distributed on $[0, k_N]$ and $k_P = 0$ (see Figure 1).[1] Then, $a = 1/2$ and some equations can be explicitly solved (see Figure 1, lower panel). In the upper panel, the vertical dotted lines separate the three possible events: excess demand (Ω_4), highly dispatched (Ω_3), and non-dispatched (Ω_1). For example, $a_4 k_N$ is exactly in the middle between the lower and the upper bound of Ω_4.



	Ω_4	Ω_3	Ω_{34}	Ω_1
Pr_c	$(D - k_H)/k_N$	k_H/k_N	D/k_N	$1 - D/k_N$
a_c	$0.5\,(D - k_H)/k_N$	$0.5\,(2D - k_H)/k_N$	$0.5\,D/k_N$	$0.5\,(k_N - D)/k_N$

Fig. 1: Illustration of probabilities for a uniform distribution (if $a_3 < a$). The probability density $f(\tilde{x}_N; k_N)$ is plotted at the upper panel (vertical dashed line).

The relation $a_4 < a_{34} < a, a_3$ holds for any proper distribution, but the relation between a_3 and a is generally inconclusive. For a uniform distribution, it can further be verified that $a_3 \Pr_3 + a_4 \Pr_4 = a_{34} \Pr_{34}$. Using this in (53) and subsequently substituting (54) and (55), yields

$$\Phi \quad = \quad \frac{a - a_{34}}{a_{34}} \left(\frac{b_N}{a} + c_N \right) + \frac{a_{34} - a_4}{a_{34}} b_H. \tag{57}$$

Thus, the correlation mark-up is strictly positive for the case of perfect correlation and a uniform distribution. With the solutions from the lower panel in Figure 1, (57) can be rearranged as

$$\Phi D = (k_N - D) \left(\frac{b_N}{a} + c_N \right) + b_H k_H. \tag{58}$$

This shows that revenues from charging scheduled demand at the level of the correlation mark-up would cover the capacity costs of highly dispatchable technologies plus the LRMC of non-dispatchable capacity above demand. The correlation mark-up is larger, if the optimum requires more highly dispatchable capacity or more non-dispatchable capacity in excess.

[1] Recall that $k_P = 0$ leads to $\Pr_2 = 0$.

2.3 Comparative Statics of $\Pr_3, \Pr_{34}, \Pr_4$

Compared to the case of independence, we now obtain a stronger result for the comparative statics of $\Pr_3$ w.r.t. k_N. This holds for any distribution f. As we can write $\tilde{x}_N = \psi k_N$ for one representative generating unit ψ, we can transform the random variable. Denote the probability density function of ψ by $f_\omega(\psi)$, which is independent from k_N. Then, $f(\tilde{x}_N; k_N) = f_\omega(\psi) \cdot \frac{1}{k_N}$. This yields

$$\begin{aligned} \Pr{}_3 &= \int_{D-k_H}^{D} f(\tilde{x}_N; k_N)\, d\tilde{x}_N \\ &= \int_{(D-k_H)/k_N}^{D/k_N} f_\omega(\psi)\, d\psi, \qquad (59) \\ \frac{\partial \Pr_3}{\partial k_N} &= f_\omega(D/k_N)\left(-\frac{D}{k_N^2}\right) - f_\omega((D-k_H)/k_N)\left(-\frac{D-k_H}{k_N^2}\right) \\ &= f(D-k_H, k_N)\, k_N \frac{D-k_H}{k_N^2} - f(D, k_N)\, k_N \frac{D}{k_N^2} \\ &= -\frac{f(D; k_N)\, D - f(D-k_H; k_N)(D-k_H)}{k_N}. \qquad (60) \end{aligned}$$

This expression is positive iff $f(D-k_H, k_N)(D-k_H) > f(D, k_N) D$. Thus, we obtain the values in Table 3.

If we additionally assume that $\tilde{x}_N$ is uniformly distributed, i.e., $f(D; k_N) = f(D-k_H; k_N)$, we obtain a closed-form result, so that $\frac{\partial \Pr_3}{\partial k_N} > 0$ and $\frac{\partial \Pr_3}{\partial D} = 0$.

	k_N	k_H	D
$\Pr_4$	$(-)$	$(-)$	$(+)$
$\Pr_{34}$	$(-)$	0	$(+)$
$\Pr_3$	$\#^1$	$(+)$	$\#^2$
$\#^1$	$(-)$ iff $f(D; k_N) D > f(D-k_H; k_N)(D-k_H)$		
$\#^2$	$(-)$ iff $f(D; k_N) < f(D-k_H; k_N)$		

Tab. 3: Comparative statics of probabilities

2.4 Comparative Statics of k_N, k_H, D

For the case of perfect correlation, unambiguous results are more difficult to obtain. Here, we concentrate on the special case of uniformly distributed marginal generating units. For parsimony, we use the shortcuts $U' = U'(D)$ and $U'' = U''(D)$ subsequently.

The comparative statics are determined from the equation system following from the total differential of the first-order conditions (50), (52), and (49), here repeated as $F = (F_N, F_H, F_D)^t = 0$

	$\partial/\partial k_N$	$\partial/\partial k_H$	$\partial/\partial D$
F_N	$-\frac{c_H k_H(2D-k_H)+c_0(D-k_H)^2}{k_N^3}$	$-\frac{(c_0-c_H)(D-k_H)}{k_N^2}$	$\frac{c_0 D-(c_0-c_H)k_H}{k_N^2}$
F_H	$-\frac{(c_0-c_H)(D-k_H)}{k_N^2}$	$-\frac{(c_0-c_H)}{k_N}$	$\frac{(c_0-c_H)}{k_N}$
F_D	$-\frac{c_0 D-(c_0-c_H)k_H}{k_N^2}$	$-\frac{(c_0-c_H)}{k_N}$	$\frac{c_0}{k_N}-U''$

	$\partial/\partial b_N$	$\partial/\partial c_N$	$\partial/\partial b_H$	$\partial/\partial c_H$	$\partial/\partial c_0$
F_N	-1	$-a$	0	$a_3\,\mathrm{Pr}_3$	$a_4\,\mathrm{Pr}_4$
F_H	0	0	-1	$-\mathrm{Pr}_4$	Pr_4
F_D	0	0	0	Pr_3	Pr_4

Tab. 4: Further partial derivatives of F_N, F_H, F_D.

with

$$F_N \quad := \quad c_H a_3 \,\mathrm{Pr}_3 + c_0 a_4 \,\mathrm{Pr}_4 - b_N - c_N a, \tag{61}$$
$$F_H \quad := \quad (c_0 - c_H)\,\mathrm{Pr}_4 - b_H, \tag{62}$$
$$F_D \quad := \quad c_H \,\mathrm{Pr}_3 + c_0 \,\mathrm{Pr}_4 - U'. \tag{63}$$

This proof becomes more complicated than the proof for the case of independence, because the Jacobian $\mathcal{J}(F)$ has no diagonal form. On the other hand, because ω is uniformly distributed, we can use the explicit expressions from Figure 1, so that, e.g.,

$$F_N \quad = \quad c_H \frac{1}{2}\frac{2D-k_H}{k_N}\frac{k_H}{k_N} + c_0 \frac{1}{2}\frac{D-k_H}{k_N}\frac{D-k_H}{k_N} - b_N - a c_N. \tag{64}$$

We obtain all derivatives in Table 4 in the same way.

For each parameter of interest, the total differential leads to a system of three equations. We solve these systems with Cramer's rule. For example, the following equation system needs to be solved with Cramer's rule to determine the comparative statics with respect to c_0:

$$-\frac{\partial F_N}{\partial c_0} \quad = \quad \frac{\partial F_N}{\partial k_N}\frac{dk_N}{dc_0} + \frac{\partial F_N}{\partial k_H}\frac{dk_H}{dc_0} + \frac{\partial F_N}{\partial D}\frac{dD}{dc_0}, \tag{65}$$
$$-\frac{\partial F_H}{\partial c_0} \quad = \quad \frac{\partial F_H}{\partial k_N}\frac{dk_N}{dc_0} + \frac{\partial F_H}{\partial k_H}\frac{dk_H}{dc_0} + \frac{\partial F_H}{\partial D}\frac{dD}{dc_0}, \tag{66}$$
$$-\frac{\partial F_D}{\partial c_0} \quad = \quad \frac{\partial F_D}{\partial k_N}\frac{dk_N}{dc_0} + \frac{\partial F_D}{\partial k_H}\frac{dk_H}{dc_0} + \frac{\partial F_D}{\partial D}\frac{dD}{dc_0}. \tag{67}$$

The determinant of the Jacobian evaluates to

$$\det(\mathcal{J}(F)) = -\frac{c_H (c_0 - c_H) D^2}{k_N^4} U'' > 0. \tag{68}$$

As we are only interested in the signs of the solutions, we can focus the further proof on the signs of the determinants of the matrices where the appropriate column of the Jacobian is replaced by the negative partial derivatives w.r.t. the parameter of interest. We obtain Table 5, where $\doteq$ denotes equivalence in the algebraic sign.

	$\partial k_N / \partial \doteq$	$\partial D / \partial \doteq$
b_N	$-\frac{(c_0-c_H)\left(-U'' k_N+c_H\right)}{k_N^2}<0$	$-\frac{c_H(c_0-c_H)D}{k_N^3}<0$
c_N	$-\frac{(c_0-c_H)\left(-U'' k_N+c_H\right)}{k_N^2}a<0$	$-\frac{c_H(c_0-c_H)D}{k_N^3}a<0$
b_H	$-\frac{(c_0-c_H)\left(U'' k_N(D-k_H)+c_H k_H\right)}{k_N^3}$	$-\frac{c_H(c_0-c_H)D}{k_N^3}k_H<0$
c_H	$-\frac{(c_0-c_H)\left[U'' k_N \frac{1}{2}\left((D-k_H)^2+D^2\right)+c_H \frac{1}{2}(2D-k_H)k_H\right]}{k_N^4}$	$-\frac{c_H(c_0-c_H)D}{k_N^3}\frac{\frac{1}{2}(2D-k_H)k_H}{k_N^2}<0$
c_0	$-\frac{(c_0-c_H)\left(-U'' k_N+c_H\right)}{k_N^2}\frac{\frac{1}{2}(D-k_H)^2}{k_N^2}<0$	$-\frac{c_H(c_0-c_H)D}{k_N^5}\frac{\frac{1}{2}(D-k_H)^2}{k_N^2}<0$

	$\partial k_H / \partial \doteq$
b_N	$-\frac{(c_0-c_H)\left(U'' k_N(D-k_H)+c_H k_H\right)}{k_N^3}$
c_N	$-\frac{(c_0-c_H)\left(U'' k_N(D-k_H)+c_H k_H\right)}{k_N^3}a$
b_H	$-\frac{c_H(c_0-c_H)k_H^2-U'' k_N\left(c_H(2D-k_H)k_H+c_0(D-k_H)^2\right)}{k_N^3}<0$
c_H	$-\frac{c_H(c_0-c_H)k_H^2(2D-k_H)-U''\left[c_H(2D-k_H)k_H+c_0\left(D^2+(D-k_H)^2\right)\right]k_N(D-k_H)}{2k_N^5}<0,$
c_0	$-\frac{(c_0+c_H)\left(U'' k_N(D-k_H)+c_H k_H\right)}{k_N^3}\frac{\frac{1}{2}(D-k_H)^2}{k_N^2}$

Tab. 5: Signs of the derivatives of k_N, k_H, D w.r.t. b_N, c_N, b_H, c_H, c_0

The ambiguous cases can be further analyzed as follows. Note that $\frac{dk_N}{db_H}, \frac{dk_H}{db_N}, \frac{dk_H}{dc_N}, \frac{dk_H}{dc_0} < 0$ iff

$$0 \quad < \quad U'' k_N (D-k_H) + c_H k_H. \tag{69}$$

Using Figure 1, we can resubstitute for $\mathrm{Pr}_3, \mathrm{Pr}_4$ to obtain

$$-U'' \quad < \quad \frac{c_H k_H}{k_N (D-k_H)} = \frac{c_H k_H / k_N}{k_N (D-k_H)/k_N} = \frac{c_H \,\mathrm{Pr}_3}{k_N \,\mathrm{Pr}_4}. \tag{70}$$

The expression on the right-hand side is increasing in k_H, but decreasing in k_N, D. We also obtain $\frac{dk_N}{dc_H} < 0$ iff

$$0 \quad < \quad U'' k_N \frac{1}{2}\left((D-k_H)^2 + D^2\right) + c_H \frac{1}{2}(2D-k_H) k_H, \tag{71}$$

which can be solved (using Figure 1) to obtain

$$-U'' \quad < \quad \frac{c_H}{k_N}\frac{\frac{1}{2}(2D-k_H)\,k_H}{\frac{1}{2}(D-k_H)^2+\frac{1}{2}D^2} = \frac{c_H}{k_N}\frac{\frac{1}{2}\frac{2D-k_H}{k_N}\frac{k_H}{k_N}}{\frac{1}{2}\frac{D-k_H}{k_N}\frac{D-k_H}{k_N}+\frac{1}{2}\frac{D}{k_N}\frac{D}{k_N}} = \frac{c_H}{k_N}\frac{a_3\,\mathrm{Pr}_3}{a_4\,\mathrm{Pr}_4+a_{34}\,\mathrm{Pr}_{34}}. \quad (72)$$

By differentiation we obtain that the right-hand side is increasing in k_H, but decreasing in k_N, D. The comparative statics for capacities and scheduled demand are summarized in Table 6.

In contrast to the case of independence, scheduled demand also depends on b_H, c_H, c_0 (via the correlation mark-up Φ). As a result, non-dispatchable and highly-dispatchable technologies can be complements with respect to b_H, c_H, c_0.

	b_N, c_N	b_H, c_H	c_0
k_N	$(-)$	$\#^4, \#^5$	$(-)$
k_H	$\#^4$	$(-)$	$\#^4$
D	$(-)$	$(-)$	$(-)$
$\#^4$	$(-)$ iff $-U''/c_H < k_H/k_N\,(D-k_H)$		
$\#^5$	$(-)$ iff $-U''/c_H < \left(D^2-(D-k_H)^2\right)/k_N\left(D^2+(D-k_H)^2\right)$		

Tab. 6: Comparative statics of capacities and scheduled demand

2.5 Costs Recovery

Comparing the case of independence and the case of perfect correlation, there is no difference in costs recovery for partially and highly dispatchable technologies: partially dispatchable technologies recover exactly costs, whereas highly dispatchable technologies fail to recover costs. Even with the correlation mark-on Φ on the price, we have $p = U'(D) = b_H + c_H\,\mathrm{Pr}_{34} = \frac{b_N}{a} + c_N + \Phi$ and thus a price below LRMC of highly dispatchable technologies.

It is more complicated for non-dispatchable technologies. The mark-up could principally improve the possibility of costs recovery. We obtain

$$E[D_N] \quad = \quad E[\tilde{x}_N|\Omega_{34}]\,\mathrm{Pr}_{34} + E[D|\Omega_1]\,\mathrm{Pr}_1, \quad (73)$$

$$E[\pi_N] \quad = \quad -(b_N/a+c_N)\,(E[\tilde{x}_N]-E[D_N]) + \Phi E[D_N]\,. \quad (74)$$

The first (negative) term in (74) represents the costs from producing more output than can be sold. The second term represents the adjustments from the correlation mark-up Φ. Thus, if $\Phi \leq 0$, non-dispatchable capacities will never recover costs. A positive correlation mark-up, however, might be sufficient to cover the loss from producing more output with non-dispatchable technologies than can be sold. Yet, the zero profit condition is only satisfied in the boundary case, where

$$\frac{\Phi}{b_N/a+c_N} \quad = \quad \frac{E\left[\tilde{x}_N\right]-E\left[D_N\right]}{E\left[D_N\right]}, \tag{75}$$

that is, if the mark-up of non-dispatchable technologies in relation to LRMC is exactly equal to the relative excess production.

If it is additionally assumed that marginal generation units are uniformly distributed, we know that $\Phi > 0$ and obtain a stronger result. Using (57), we can rewrite (74) as follows:

$$\begin{aligned}
E\left[\pi_N\right] &= -\left(\frac{b_N}{a}+c_N\right)\left(E\left[\tilde{x}_N\right]-E\left[D_N\right]\right)+\Phi E\left[D_N\right] \\
&= -\left(\frac{b_N}{a}+c_N\right)E\left[\tilde{x}_N\right]+\left(\frac{b_N}{a}+c_N\right)E\left[D_N\right] \\
&\quad +\left(\frac{b_N}{a}+c_N\right)\frac{a-a_{34}}{a_{34}}E\left[D_N\right]+b_H\frac{a_{34}-a_4}{a_{34}}E\left[D_N\right] \\
&= -\left(\frac{b_N}{a}+c_N\right)E\left[\tilde{x}_N\right]+\left(\frac{b_N}{a}+c_N\right)\frac{a}{a_{34}}E\left[D_N\right]+b_H\frac{a_{34}-a_4}{a_{34}}E\left[D_N\right] \\
&= -\left(\frac{b_N}{a}+c_N\right)\frac{a}{a_{34}}a_{34}k_N+\left(\frac{b_N}{a}+c_N\right)\frac{a}{a_{34}}E\left[D_N\right]+b_H\frac{a_{34}-a_4}{a_{34}}E\left[D_N\right] \\
&= \left(\frac{b_N}{a}+c_N\right)\frac{a}{a_{34}}\left(E\left[D_N\right]-a_{34}k_N\right)+b_H\frac{a_{34}-a_4}{a_{34}}E\left[D_N\right] \\
&= \frac{a}{a_{34}}\left(\frac{b_N}{a}+c_N\right)\left(E\left[D_N\right]-a_{34}k_N\right)+\frac{a_{34}-a_4}{a_{34}}b_H E\left[D_N\right].
\end{aligned} \tag{76}$$

This is positive because $a_{34}-a_4=\frac{1}{2}\frac{k_H}{k_N}>0$ and

$$\begin{aligned}
E\left[D_N\right]-a_{34}k_N &= E\left[\tilde{x}_N|\Omega_{34}\right]\Pr_{34}+D\Pr_{1}-E\left[\tilde{x}_N|\Omega_{34}\right] \\
&= \left(D-E\left[\tilde{x}_N|\Omega_{34}\right]\right)\Pr{}_1 \\
&= \frac{1}{2}D\Pr{}_1>0.
\end{aligned} \tag{77}$$

To conclude, the correlation mark-up might be sufficient to cover the loss from excess output during Ω_1. Then, depending on the shape of the utility function and the distribution of $\tilde{x}_N$, the price signal leads either to strictly positive profits or to losses for non-dispatchable capacities. However, the main result from Eisenack and Mier (2019) holds: markets cannot be designed in a conventional way as soon as non-dispatchable technologies enter the market. For a detailed analysis of costs recovery under the case of perfect correlation see Mier (2018).

2.6 Comparison and Extensions

Eisenack and Mier (2019) discuss three model extensions, from whom one is the focus of this technical appendix itself, and compare their model with the standard literature. The proof for multiple periods is equivalent for both cases (see Section 1.6). It remains to show that the results hold when comparing with the standard model without dispatchability types, and under the assumption of downward-dispatchability.

Peak-load pricing without dispatchability types. The first-order conditions are

$$-\frac{\partial E[J]}{\partial D} = c_0 \Pr_4 + c_H \Pr_3 + c_P \Pr_2 - U'(D), \tag{78}$$
$$\frac{\partial E[J]}{\partial k_H} = c_0 \Pr_4 - c_H \Pr_4 - b_H, \tag{79}$$
$$\frac{\partial E[J]}{\partial k_P} = c_0 \Pr_4 + c_H \Pr_3 - c_P \Pr_{34} - b_P, \tag{80}$$
$$\frac{\partial E[J]}{\partial k_N} = c_0 a_4 \Pr_4 + c_H a_3 \Pr_3 + c_P a_2 \Pr_2 - c_N a - b_N. \tag{81}$$

Setting the first order conditions to zero and solving the system yields $U'(D) = b_P + c_P \Pr_{234}$ with

$$\Pr_4 = \frac{b_H}{c_0 - c_H}, \tag{82}$$
$$\Pr_{34} = \frac{b_P - b_H}{c_H - c_P}, \tag{83}$$
$$\Pr_{234} = \frac{b_N + c_N a}{c_P a_2} - \frac{c_H a_3 - c_P a_2}{c_P a_2 (c_H - c_P)} b_P - \frac{(c_H - c_P)(c_0 a_4 - c_P a_2) - (c_H a_3 - c_P a_2)(c_0 - c_P)}{c_P a_2 (c_H - c_P)(c_0 - c_H)} b_H. \tag{84}$$

Under certain costs constellations, which are again not a boundary case, we have $\Pr_4 < \Pr_{34} < \Pr_{234}$.

Downward-dispatchability. The relevant derivatives are

$$\frac{\partial E[J]}{\partial k_N} = (c_H - c_N) a_3 \Pr_3 + (c_0 - c_N) a_4 \Pr_4 - b_N. \tag{85}$$
$$\frac{\partial E[J]}{\partial k_P} = c_N \Pr_{12} + c_H \Pr_3 + c_0 \Pr_4 - b_P - c_P, \tag{86}$$

As in Section 2.1, setting the two expressions to zero and using $\Pr_{12} = 1 - \Pr_3 - \Pr_4$ yields an overdetermined equation system (two equations for $\Pr_3$). This can only be solved for a boundary case with specific cost parameters.

References

Chao, H.-P. (1983). Peak load pricing and capacity planning with demand and supply uncertainty. *The Bell Journal of Economics 14*(1), 179–190.

Chao, H.-P. (2011). Efficient pricing and investment in electricity markets with intermittent resources. *Energy Policy 39*(7), 3945–3953.

Eisenack, K. and M. Mier (2018). Peak-load pricing with different types of dispatchability. *Oldenburg Discussion Papers in Economics V-411-18*.

Eisenack, K. and M. Mier (2019). Peak-load pricing with different types of dispatchability. *Journal of Regulatory Economics*, DOI: XXXXX.

Mier, M. (2018). Policy implications of a world with renewables, limited dispatchability, and fixed load. *Oldenburg Discussion Papers in Economics V-412-18, revised version 2019 "Efficient pricing of electricity revisited"*.