**Microeconomics**
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Sheets 1: Chapter 3, Preference and Utility

## Outline

Part II: Choice and Demand

1. **Preference and Utility**
   - **The concept of a consistent utility order ("axioms of rational choice")**
   - **Indifference curves (graphical representation of utility functions in two-dimensional space)**
   - **Utility functions**
2. *Utility Maximization and Choice*
3. *Income and Substitution Effects*
4. *Demand Relationships among Goods*



## Axioms of Rational Choice

- "Axioms": basic assumptions which are considered to require no proof as they are obvious
- 3 axioms
  - Completeness
  - Transitivity
  - (Continuity) technical assumption



## Axioms of Rational Choice

- Completeness
  - A > B (A is preferred to B)
  - or
  - B > A (B is preferred to A)
  - or
  - A ≈ B (A and B are equally attractive)

  - Completeness: for each potential choice exactly one of the three is true.
  - Plausible?



## Axioms of Rational Choice

- Transitivity
  - If A > B
  - and
  - B > C
  - then
  - A > C

  - Plausible?
  - Could you think about other situations?



## Utility

- As a result of completeness and transitivity people can rank all possible situations in order from the least desirable to the most
- And then attach the term "utility" to this ranking
  - With a higher utility representing a higher desirability
  - And a lower utility representing a lower desirability
- We can now attach an arbitrary set of numbers to such an utility ranking which accurately reflects the order

## Utility

- For A > B > C
- For example:
  - U(A) = 10, U(B) = 5, U(C) = 1
- Or:
  - U(A) = 1000, U(B) = 100, U(C) = 1
- Only the order matters, nothing can be said about the distance between different situations
  - An "ordinal" utility order



## Utility

- An ordinal utility order expressed mathematically:
  - Any numerical utility ranking (U) can be replaced by any transformation F(U)
    - If F(U) is strictly increasing
    - Wenn F(U) eine monoton steigende Funktion ist
    - In other words: if F'(U) > 0 for all U
  - Example:
    - $F(U) = U^2$
    - F'(U) = 2U



## Utility

- Alternatively, a cardinal utility order allows for statements such as U(A) = 2 U(B)
- A cardinal utility order expressed mathematically:
  - Any cardinal utility ranking (U) can be replaced by any transformation F(U)
    - If F(U) is a linearly increasing function
    - Wenn F(U) eine linear steigende Funktion ist
    - In other words: if F'(U) > 0 and constant for all U
  - Example:
    - F(U) = 2U; F'(U) = 2
  - Real world examples for cardinal and ordinal orders?



## Utility from the Consumption of Goods

- n goods $x_1, x_2,\ldots, x_n$ , with x representing the quantity of each good
- A utility function….
  - utility = $U(x_1, x_2,\ldots, x_n)$ ???
  - utility = $U(x_1, x_2,\ldots, x_n$; other things)
  - What allows us to simplify from "other things"?
    - The ceteris paribus assumption
    - utility = $U(x_1, x_2,\ldots, x_n)$ !
  - Notation throughout the book if only two goods are considered:
    - utility = U(x, y)



## Towards an indifference curve…

- Basic assumption: "goods are good"!
  - Non-satiation
  - Plausible?
- Graph on the blackboard (Figure 3.1 in the book on page 73)
- What does this imply for the indifference curve?
  - [Indifferenzkurve, Isonutzenlinie]
  - [definition: an indifference curve shows a set of consumption bundles among which the individual is indifferent]
  - The indifference curves slopes down!
  - (can it be horizontal?)



## Indifference curves

- The indifference curve displays the willingness of individuals to trade products against each other…
- Graph on the blackboard (Figure 3.2 in the book on page 74)
  - Slope = negative (goods are good….)
  - Slope is increasing ("-" → "0")
  - Or decreasing in absolute terms: "it is getting flatter"
  - Is that plausible? Try to defend this slope verbally!
  - Marginal Rate of Substitution
  - (Grenzrate der Substitution)

$$MRS_{yx} = -\left.\frac{dy}{dx}\right|_{U=U_1}$$

## Indifference curves

- How many indifference curves are there?
  - Graph on the blackboard (Figure 3.3 on page 75)
- Can indifference curves intersect?
  - Graph on the blackboard (Figure 3.4 on page 76)



## Indifference curves

- An alternative way to describe a diminishing MRS:
  - All points which are preferred to an indifference curve form a convex set
  - ≈
  - The indifference curve is strictly convex to the origin
  - ≈
  - Any line which combines two points on the indifference curve is in the convex set
  - ≈
  - $U(\lambda(x_1,y_1) + (1-\lambda)(x_2,y_2)) > U(x_1,y_1) = U(x_2,y_2)$
  - Graph on the blackboard (Figure 3.5 on page 77)



## Indifference curves

- Is this plausible?
  - Think about cake and olives…
  - What's about bread and beer?
- The strict convexity of the indifference curve is an assumption
  - As is its negative slope
  - As is the fact that is has no kinks (it is "differentiable")
- Assumptions of convexity and no kinks are made for technical reasons – as we will see later we don't have clearly defined demand curves otherwise
- Show overhead on utility functions



## Indifference curves

- We had a graphical and a verbal explanation of the MRS:

$$MRS_{yx} = -\left.\frac{dy}{dx}\right|_{U=U_1}$$

- We now derive the MRS mathematically (on the blackboard, formulas 3.16 and 3.17 in the book, page 80)

$$MRS_{yx} = -\left.\frac{dy}{dx}\right|_{U=U_1} = \frac{\partial U / \partial x}{\partial U / \partial y}$$



## Indifference curves

- Is this logical?

$$MRS_{yx} = -\left.\frac{dy}{dx}\right|_{U=U_1} = \frac{\partial U / \partial x}{\partial U / \partial y}$$

- If the marginal utility of x is twice as high as of y, how much y would you give for an x?



## Indifference curves

- We go for a moment beyond the text of the book and treat something which appears later in Chapter 12
- What would two individuals (call them hungry and thirsty) do, if their MRS for two products, say beer and döner, differs?
- On the blackboard…..

## Examples of Utility Functions

- Cobb-Douglas utility function:
  - $U(x, y) = x^{\alpha} y^{\beta}$
  - How do the indifference curves look like?
  - What is the $MRS_{yx}$?

$$MRS_{yx} = -\left.\frac{dy}{dx}\right|_{U=U_1} = \frac{\partial U / \partial x}{\partial U / \partial y}$$

- Find on the blackboard:

$$MRS_{yx} = \frac{\alpha}{\beta}\frac{y}{x}$$



## Examples of Utility Functions

- Cobb-Douglas utility function – how does this graph?

$$MRS_{yx} = \frac{\alpha}{\beta}\frac{y}{x}$$

- On the blackboard (Figure 3.8(a), page 83)
- Diminishing MRS
- Multiplicative link of arguments: if one product is zero, utility is zero
- α β are weighting factors



## Examples of Utility Functions

- Perfect substitutes:
  - U (x, y) = α x + β y
  - What is the $MRS_{yx}$?
    - α/β
  - MRS is a constant
- How does this graph? On the blackboard
- Examples?
  - Fuel from Shell or Esso?



## Examples of Utility Functions

- Perfect complements:
  - U (x, y) = min (α x, β y)
  - Goods only provide utility in combination
  - The scarce good determines the utility level
  - Graph on blackboard
- How does this graph? On the blackboard
- Examples?
  - Left and right shoes
  - U = min (1 L, 1 R)
  - 4 left shoes and no right one: utility zero



## Examples of Utility Functions

- CES (constant elasticity of substitution) utility function:

$$U(x,y) = \frac{\alpha x^{\delta}}{\delta} + \frac{\beta y^{\delta}}{\delta}$$

- A more general functional form, which allows for the depiction of the CD case as well as perfect substitutes and complements.
- Derive MRS on the blackboard $MRS_{yx} = \frac{\alpha}{\beta}\left(\frac{x}{y}\right)^{(\delta-1)}$
- With σ = elasticity of substitution = 1/(1-δ)



## Examples of Utility Functions

- CES utility function:
  - Derive MRS for δ =1, δ = 0, δ = -∞
- Have fun with a few alternative indifference curves on the blackboard (Varian, Figures 3.5-3.7)
- Homothetiy:
  - $MRS_{yx}$ only depends on the ratio y/x, not on the level of y and x
  - Show this on the blackboard
  - Show that the CD function is homothetic on the blackboard (example 3.3 in the book on page 86)