Microeconomics
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Harald Grethe
Department of International Agricultural Trade and Development

Sheets 3: Chapter 5, Income and Substitution Effects

Outline

Part II: Choice and Demand

1. *Preference and Utility*
2. *Utility Maximization and Choice*
3. **Income and Substitution Effects**
   - **Effects of a Change in Income (Graphically)**
   - **Effects of a Change in Own Price (Graphically)**
   - **Graphical Derivation of the Demand Curve**
   - **Graphical Deriv. of the Compensated Demand Curve**
   - **Mathematical Derivation of the Price Response**
   - **Elasticities**
   - **Consumer Surplus/Compensating Variation**
4. *Demand Relationships among Goods*

To Start with: Demand Functions

- $x_i^* = x_i(p_1, p_2, \ldots, p_n, I)$
- or $x^* = x(p_x, p_y, I)$ and $y^* = y(p_x, p_y, I)$
- Homogeneity (Homogenität)?
- Think graphically….
- Demand functions are homogeneous of degree zero (homogen vom Grade null) in income and prices
- Generally:
  - $x_i^* = x_i(p_1, p_2, \ldots, p_n, I) = x_i(tp_1, tp_2, \ldots, tp_n, tI)$
  - For our Cobb Douglas case?
- Freedom of money illusion (Freiheit von Geldillusion)

Effects of a Change in Income (Graphically)

- $x^* = x(p_x, p_y, I)$
- We look at $\frac{\partial x}{\partial I}$

$$\frac{\partial x}{\partial I} \geq 0?$$

$$\frac{\partial x}{\partial I} \leq 0?$$

Effects of a Change in Income (Graphically)

- An increase in income for normal goods: graph on the blackboard (Figure 5.1 in the book on page 124)

If $\frac{\partial x}{\partial I} \geq 0$ over some range, x is a normal good over that range

- An increase in income for a normal and an inferior good: graph on the blackboard (Figure 5.2 in the book on page 125)

If $\frac{\partial x}{\partial I} < 0$ over some range, x is an inferior good over that range

Effects of a Change in Own Price (Graphically)

- A fall in the price of x: graph on the blackboard (Figure 5.3 in the book on page 126)
  - x is increasing for two reasons:
    - The substitution effect (Substitutionseffekt)
      - Along the indifference curve
    - The income effect (Einkommenseffekt)
      - "Jump" to a higher indifference curve, as the fall in price implies a higher real income
  - For a normal good, the substitution and the income effect work in the same direction $\frac{\partial x}{\partial p_x} < 0$

Effects of a Change in Own Price (Graphically)

- An increase in price: Figure 5.4 in the book (p. 127)
- An increase in the price of x being an inferior good: graph on the blackboard
  - For an inferior good, the substitution and the income effect work in the opposite direction

Effects of a Change in Own Price (Graphically)

- An increase in the price of x being a "Giffen" good: graph on the blackboard
  - For a Giffen good, the substitution and the income effect work in the opposite direction
    - and the income effect exceeds the substitution effect such that

$$\frac{\partial x}{\partial p_x} > 0$$

Deriving the Demand Curve Graphically

- Graph on the blackboard (Figures 5.5 to 5.7 in the book on pages 129-133)
- Normal demand curve: $x = x(p_x, \overline{p}_y, \overline{I})$
  - "Marshallian" demand curve
  - Can be observed in the market
- Compensated demand curve $x^c = x(p_x, \overline{p}_y, \overline{u})$
  - "Hicksian" demand curve
  - cannot be observed
  - Is steeper than the normal demand curve for superior ("normal") goods
  - Useful for welfare analysis

Deriving the Demand Curve Graphically

- Why we don't like concave or kinked indifference curves: graphs on the blackboard
- A mathematical development of demand quantity response to price changes: the Slutsky equation (on the blackboard, pages 135-137)
- Derivation of the compensated demand curve
  - The compensated demand curve x_c is the first order derivative of the consumer expenditure function with respect to the price of x

Elasticities – Why Do We Use Them? An Example

x = 80 + 0.02 I (UK: £, kg) $\frac{dx}{dI} = 0.02$

I = 1,000 ⇒ x = 80 + 0.02·1,000 = 100

I = 1,100 ⇒ x = 80 + 0.02·1,100 = 102

x = 80.000 + 10 I (Germany: €, g) $\frac{dx}{dI} = 10$

I = 2,000 ⇒ x = 80.000 + 10·2,000 = 100,000

I = 2,200 ⇒ x = 80.000 + 10·2,200 = 102,000

- Parameters can't be compared because units of measurement are different.

Example (2)

- x = 80 + 0.02 I (UK: £, kg), I = 1,000, x = 100

  dI = 100, dx = 2
- Calculation of income elasticity of demand for UK:

$$\varepsilon_{x,I} = \frac{dx/x}{dI/I} = \frac{dx}{dI}\frac{I}{x} = \frac{2}{100}\frac{1,000}{100} = 0.2; \quad \text{for } I = 1,000$$

- The elasticity of a variable "a" with respect to a variable "b" is the percentage change in variable "a" given a one percent change in variable "b"

Elasticities: Summary

- Measure of the intensity of reaction
- Advantage versus coefficients
  - Comparable with different functional forms
  - Comparable with different units of measurement
  - Easily to grasp intuitively
- Example: income elasticity $\varepsilon_{x,I} = \frac{dx/x}{dI/I} = \frac{dx}{dI}\frac{I}{x}$
  - The percentage change in demand quantity given a one percent change in income

Elasticities: Summary (2)

- Calculation of elasticity from first derivative:

$$\text{UK}: \varepsilon_{x,I} = 0.02\frac{1000}{100} = 0.2 \quad \text{for } I = 1{,}000$$

$$\text{Germany}: \varepsilon_{x,I} = 10\frac{2.000}{100.000} = 0.2 \quad \text{for } I = 2{,}000$$

- With a linear functional form, the elasticity changes along the curve
- Other functional forms exist, for example the isoelastic:

$$x = 25\ I^{0.2}$$

Demand Elasticities

- Which elasticities do we have for
  - **x = x (P_x, P_y, I)**
- Own price elasticity of demand (of "quantity demanded")

$$\varepsilon_{x,p_x} = \frac{dx}{dp_x}\frac{p_x}{x}$$

$\varepsilon_{x,p_x} < -1$, (or $> |-1|$) : demand for x is "elastic"

$\varepsilon_{x,p_x} = -1$: demand for x is "unit - elastic"

$\varepsilon_{x,p_x} > -1$, (or $< |-1|$) : demand for x is "inelastic"

$\varepsilon_{x,p_x} > 0$: x is a Giffen good

Demand Elasticities (2)

- Which elasticities do we have for
  - **x = x (P_x, P_y, I)**
- Cross price elasticity of demand for x with respect to the price of y

$$\varepsilon_{x,p_y} = \frac{dx}{dp_y}\frac{p_y}{x}$$

$\varepsilon_{x,p_y} < 0$: x and y are "gross complements"

$\varepsilon_{x,p_y} > 0$: x and y are "gross substitutes"

Demand Elasticities (3)

- Which elasticities do we have for
  - **x = x (P_x, P_y, I)**
- Income elasticity of demand for x

$$\varepsilon_{x,I} = \frac{dx}{dI}\frac{I}{x}$$

$\varepsilon_{x,I} > 1$: x is a "luxury" ("Luxusgut")

$0 < \varepsilon_{x,I} < 1$: x is a "necessity" ("Sättigungsgut")

$\varepsilon_{x,I} < 0$: x is an "inferior good" ("inferiores Gut")

Demand Elasticities (4)

- And which elasticities do we have for
  - **x^c = x (P_x, P_y, u)**

$\varepsilon_{x^c,p_x} = \frac{dx^c}{dp_x}\frac{p_x}{x}$, compensated own price elasticity of demand for x

$\varepsilon_{x^c,p_y} = \frac{dx^c}{dp_y}\frac{p_y}{x}$, compensated cross price elasticity of demand for x

- And how do they differ from the uncompensated price elasticities?
  - On the blackboard (page 140-141 in the book)

Relationships among Elasticities

- Homogeneity of demand functions:
  - The sum of the income elasticity and all uncompensated price elasticities for one good must be zero
  - On the blackboard

Relationships among Elasticities (2)

- The "Engel Aggregation" (or "Adding Up")
  - Marginal expenditure shares must add up to one
  - In other words:
  - The weighted (by expenditure shares) average on income elasticities for all goods that a person buys must add up to one
  - On the blackboard
- We don't deal with the "Cournot Aggregation"

Price Elasticity and Total Spending

- What is the relationship between the own price elasticity of a good and the effect of a own price change on total spending on that good?
- On the blackboard

$$\frac{\partial E_X}{dp_x} < 0 \text{ if } \varepsilon_{x,p_x} < -1,$$

if demand is elastic, expenditure decreases with an increasing price

$$\frac{\partial E_X}{dp_x} > 0 \text{ if } \varepsilon_{x,p_x} > -1,$$

if demand is inelastic, expenditure increases with an increasing price

Consumer Surplus and Compensating Variation

- The standard concept of consumer surplus
  - Graph on the blackboard
- A more precise welfare measure: the compensating variation (CV)
  - Algebraically: $CV = E(p_x^1, p_y, u) - E(p_x^0, p_y, u)$
  - Graphically: $CV = \int_{p_x^0}^{p_x^1} dE = \int_{p_x^0}^{p_x^1} x^c(p_x^1, p_y, u)\, dp_x$
    - On the blackboard!
    - When does the difference matter?

Price Indices as a Welfare Measure?

- Consumer price indices (Verbraucherpreisindex): Paasche and Laspeyres (on the blackboard)
- Substitution bias:
  - The welfare reduction resulting from a price increase as measured by the Laspeyres CPI overestimates the real welfare effect
  - The welfare increase resulting from a price decrease as measured by the Laspeyres CPI underestimates the real welfare effect