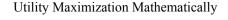


# Utility Maximization Graphically

## The two-good case graphically

- The budget constraint: graph on the blackboard (Figure 4.1 in the book on page 96)
  - $\hfill\square$  Slope and intersections with product axes
- Utility maximization: graph on the blackboard (Figure 4.2 in the book on page 97)
- Second order conditions: graph on the blackboard (Figure 4.3 in the book on page 98)
- Corner solutions: graph on the blackboard (Figure 4.4 in the book on page 99)

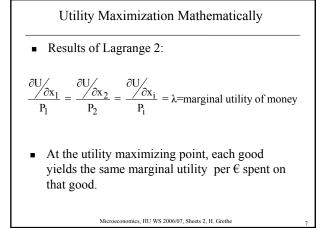
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- The n-good case mathematically
  - Applying the Lagrange method: on the blackboard (in the book on page 100-101; basics on pages 38-40)
  - Results of Lagrange:

$$\frac{\frac{\partial U}}{\partial x_{i}}}{\frac{\partial U}}{\frac{\partial x_{i}}{\partial x_{i}}} = \frac{P_{i}}{P_{j}} = -\frac{dx_{j}}{dx_{i}} = MRS_{x_{j}}x_{i}$$

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Utility Maximization Mathematically

Example: deriving demand fuctions from a Cobb Douglas utility function: on the blackboard (Example 4.1 on pages 102-103 in the book)

Result if 
$$\alpha + \beta = 1$$
:  
 $x = \alpha \frac{I}{P_x}$ ,  $y = \beta \frac{I}{P_y}$ 

- What kind of demand response is this?
  - x does not depend on  $P_y$
  - Budget shares are constant:  $P_x x/I = \alpha$ • □ Is this realistic? (overhead sheets)

## Utility Maximization Mathematically

- Example: demand functions from a CES utility function (we skip example 4.2 in the book)
  - We show that budget shares are not constant in case of changing prices on the blackboard
  - Still: budget shares do not change with increasing income (homothetic function)/income elasticities = 1
- There are more sophisticated demand systems which allow for income elasticities differing from one:
  - Linear Expenditure System (LES) •
  - Almost Ideal Demand System (AIDS)
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## The Indirect Utility Function

- Why?
  - · Welfare analysis
    - To derive the consumer expenditure function
    - To derive the compensated demand curve

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#### The Indirect Utility Function

- Direct utility function (direkte Nutzenfunktion):  $U = U(x_1, x_2)$
- $\begin{array}{l} x_2, \ldots, x_n \\ x_1^* = x_1 \ (p_1, p_2, \ldots, p_n, I) \\ \bullet & \text{With } x_1^* \text{ being the utility maximizing quantity of } x_1 \text{ under a given set of prices and income.} \end{array}$
- Maximum utility =  $U(x_1^*, x_2^*, ..., x_n^*)$
- $= V(p_1, p_2, ..., p_n, I)$
- which is the indirect utility function (indirekte Nutzenfunktion) implying a utility maximization process
- Exchange dependent and independent variable, assign the term "E"(xpenditure) for "I"(ncome), and call a specific utility level "U":
- $E = E(p_1, p_2, ..., p_n, U)$ 
  - which is the consumer expenditure function (Ausgabenfunktion)

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# The Indirect Utility Function

- "Duality": Many constrained maximization problems have associated "dual" constrained minimization problems
- Max U(U( $x_1, x_2,..., x_n$ ) Min E =  $x_1p_1 + x_2x_2...+x_np_n$ • s.t.  $I = x_1p_1 + x_2x_2... + x_np_n$  • s.t.  $\overline{U} = U(x_1, x_2, ..., x_n)$ 
  - Equivalence is obvious if looked at in a graph (blackboard)

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## Applying the Expenditure Function for Welfare Analysis

- The idea of compensating a change in a price by a change in income
  - An example on the blackboard (example 4.3, case 1, on page 107 and example 4.4, case 1, on page 110 in the book.
  - The "lump sum principle":
    - Taxes on a person's general income reduce utility to a lesser extent than taxes on specific goods
    - General income grants to poor people will raise utility more than a similar amount of money spent on subsidizing specific goods

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# Properties of Expenditure Functions

 $\geq 0$ 

• Non-decreasing in prices:  $\partial E$  (for all goods)

- Homogeneous of degree one in prices
  - doubling all prices doubles expenditures

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# Properties of Expenditure Functions

- Homogeneity:
  - Important concept in supply and demand systems
  - $y = f(x_1, x_2, ..., x_n)$
  - $\alpha^{r} y = f(\alpha x_{1}, \alpha x_{2}, ..., \alpha x_{n})$
  - $\square$  **r** = degree of homogeneity
- Examples on the blackboard
- What about our expenditure, indirect utility and utility functions?

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