Microeconomics
WS 2006/07

Harald Grethe
Department of International Agricultural Trade and Development

Sheets 2: Chapter 4, Utility Maximization and Choice

Outline

Part II: Choice and Demand

1. *Preference and Utility*
2. **Utility Maximization and Choice**
   - **Optimization verbally**
   - **Optimization graphically**
   - **Optimization mathematically**
   - **The concepts of the indirect utility and consumer expenditure functions**
     - **Homogeneity**
3. *Income and Substitution Effects*
4. *Demand Relationships among Goods*

Utility Maximization Verbally

- Basic assumption: individuals behave as utility maximizers
  - Do we agree?
  - By assumption, we defined marginal utility as positive (goods are good). What does "maximization" mean in this context?
- Income restriction
  - All income spent
- Optimization rule:
  - Psychological rate of trade off = market rate of trade off

$$\mathbf{MRS_{yx} = P_x/P_y}$$

Utility Maximization Verbally

- An example
  - **MRS_{yx} = 2**
    - (x has double the marginal utility of y)
  - And **P_x/P_y = 1**
  - What should the individual do?
  - Buy x, sell y!
  - What happens?
    - **MRS_{yx}** ↓
    - **P_x/P_y** ↑
    - Until?
    - **$MRS_{yx} = P_x/P_y$** (Logical reasoning, no proof)

Utility Maximization Graphically

- The two-good case graphically
  - The budget constraint: graph on the blackboard (Figure 4.1 in the book on page 96)
    - Slope and intersections with product axes
  - Utility maximization: graph on the blackboard (Figure 4.2 in the book on page 97)
  - Second order conditions: graph on the blackboard (Figure 4.3 in the book on page 98)
  - Corner solutions: graph on the blackboard (Figure 4.4 in the book on page 99)

Utility Maximization Mathematically

- The n-good case mathematically
  - Applying the Lagrange method: on the blackboard (in the book on page 100-101; basics on pages 38-40)
  - Results of Lagrange:

$$\frac{\partial U / \partial x_i}{\partial U / \partial x_j} = \frac{P_i}{P_j} = -\frac{dx_j}{dx_i} = MRS_{x_j x_i}$$

Utility Maximization Mathematically

- Results of Lagrange 2:

$$\frac{\partial U / \partial x_1}{P_1} = \frac{\partial U / \partial x_2}{P_2} = \frac{\partial U / \partial x_i}{P_i} = \lambda = \text{marginal utility of money}$$

- At the utility maximizing point, each good yields the same marginal utility per € spent on that good.

Utility Maximization Mathematically

- Example: deriving demand fuctions from a Cobb Douglas utility function: on the blackboard (Example 4.1 on pages 102-103 in the book)
- Result if $\alpha + \beta = 1$: $x = \alpha \frac{I}{P_x}$, $y = \beta \frac{I}{P_y}$
- What kind of demand response is this?
  - x does not depend on P_y
  - Budget shares are constant: P_x x/I = α
    - Is this realistic? (overhead sheets)
  - Income elasticity = 1
  - Own price elasticity = -1

Utility Maximization Mathematically

- Example: demand functions from a CES utility function (we skip example 4.2 in the book)
  - We show that budget shares are not constant in case of changing prices on the blackboard
  - Still: budget shares do not change with increasing income (homothetic function)/income elasticities = 1
- There are more sophisticated demand systems which allow for income elasticities differing from one:
  - Linear Expenditure System (LES)
  - Almost Ideal Demand System (AIDS)

The Indirect Utility Function

- Why?
  - Welfare analysis
    - To derive the consumer expenditure function
    - To derive the compensated demand curve

The Indirect Utility Function

- Direct utility function (direkte Nutzenfunktion): $U = U(x_1, x_2, \ldots, x_n)$
- $x_1^* = x_1 (p_1, p_2, \ldots, p_n, I)$
  - With x_1^* being the utility maximizing quantity of x_1 under a given set of prices and income.
- Maximum utility = $U(x_1^*, x_2^*, \ldots, x_n^*)$
- $= V(p_1, p_2, \ldots, p_n, I)$
  - which is the indirect utility function (indirekte Nutzenfunktion) implying a utility maximization process
- Exchange dependent and independent variable, assign the term "E"(xpenditure) for "I"(ncome), and call a specific utility level "U":
- $E = E(p_1, p_2, \ldots, p_n, U)$
  - which is the consumer expenditure function (Ausgabenfunktion)

The Indirect Utility Function

- "Duality": Many constrained maximization problems have associated "dual" constrained minimization problems

- Max $U(U(x_1, x_2, \ldots, x_n)$
  - s.t. $I = x_1p_1 + x_2x_2 \ldots + x_np_n$
- Min $E = x_1p_1 + x_2x_2 \ldots + x_np_n$
  - s.t. $\overline{U} = U(x_1, x_2, \ldots, x_n)$

- Equivalence is obvious if looked at in a graph (blackboard)

Applying the Expenditure Function for Welfare Analysis

- The idea of compensating a change in a price by a change in income
  - An example on the blackboard (example 4.3, case 1, on page 107 and example 4.4, case 1, on page 110 in the book.
  - The "lump sum principle":
    - Taxes on a person's general income reduce utility to a lesser extent than taxes on specific goods
    - General income grants to poor people will raise utility more than a similar amount of money spent on subsidizing specific goods

Properties of Expenditure Functions

- Non-decreasing in prices: (for all goods) $$\frac{\partial E}{\partial p_i} \geq 0$$
- Concave in prices: graph on blackboard (Figure 4.7 on page 113 in the book)
- Homogeneous of degree one in prices
  - doubling all prices doubles expenditures

Properties of Expenditure Functions

- Homogeneity:
  - Important concept in supply and demand systems
  - $\mathbf{y = f(x_1, x_2, \ldots, x_n)}$
  - $\boldsymbol{\alpha^r y = f(\alpha x_1, \alpha x_2, \ldots, \alpha x_n)}$
    - **r** = degree of homogeneity
- Examples on the blackboard
- What about our expenditure, indirect utility and utility functions?