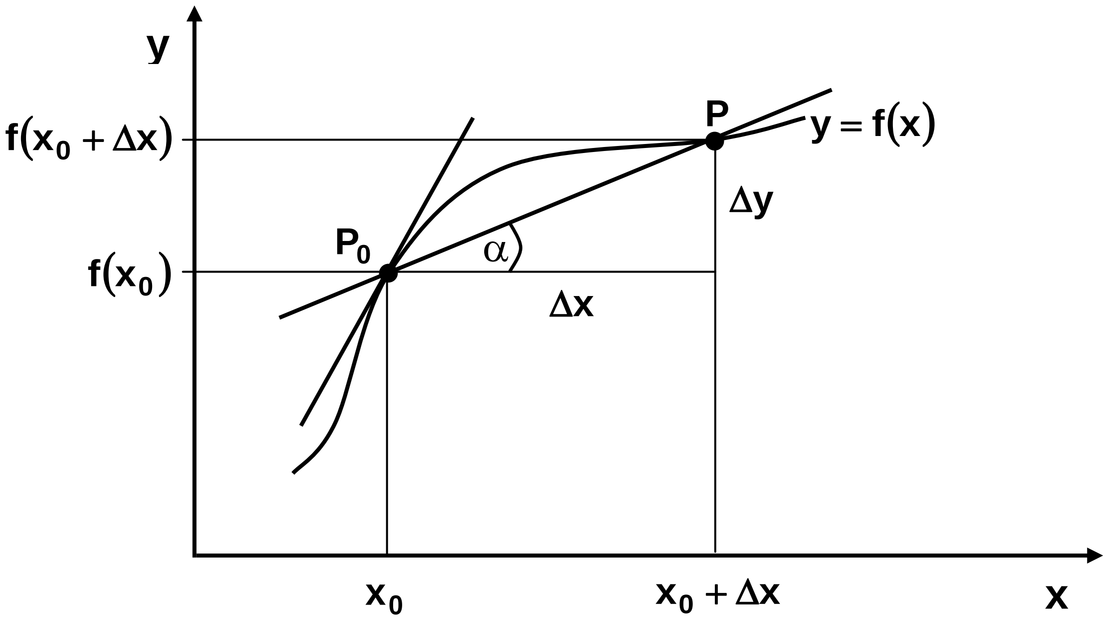
2.2 Differential calculus, integral calculus

Concerning many economical and other practical questions it will be of interest to know about the change behaviour of functions.



Difference quotient : $\frac{\Delta y}{\Delta x} = \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$

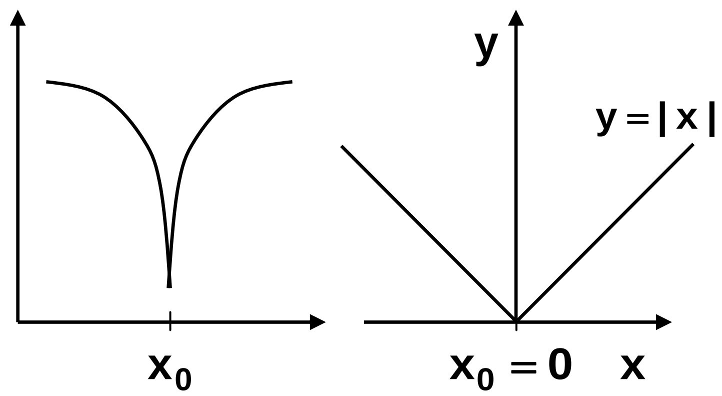
Transition to the limit value (from right hand and left hand)

Does the differential quotient

$\lim\limits_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$ exist ?

no

f is not differentiable in x_0



yes

f is differentiable in x_0

The limit value is called derivative of the function f in point x_0

with the designation $f'(x_0)$

(gradient of the function in x_0;

gradient of the tangent to the function $f(x)$ in point x_0).

The function $y = f(x)$ is called differentiable, if $f(x)$ is differentiable in all points of the definition domain.

The derivatives in all points generate a function of the variable x, which is designated with $f'(x)$ or shortly f' (following Lagrange), respectively with $\frac{dy}{dx}$ (after Leibnitz).

Technology of differentiation:

Derivative of basic functions		
$f(x)$	$f'(x)$	
x^n	$n \cdot x^{n-1}$	$\begin{cases} a) & n \in N,\ x \in R \\ b) & n \in G,\ x \neq 0 \\ c) & n \in R,\ x > 0 \end{cases}$
e^x	e^x	
$\ln x$	$\frac{1}{x}$	
$\sin x$	$\cos x$	
$\cos x$	$-\sin x$	
a^x	$a^x \cdot \ln a$	
$\log_a x$	$\frac{1}{x \cdot \ln a}$	

+

Rule of derivation	
Sum rule	$[f(x) \pm g(x)]' = f'(x) \pm g'(x)$
Product rule	$[f(x) \cdot g(x)]' = f'(x) \cdot g(x) + f(x) \cdot g'(x)$
Quotient rule	$\left[\frac{f(x)}{g(x)}\right]' = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x))}{(g(x))^2}$
Chain rule	$[g(f(x))]' = g'(f(x)) \cdot f'(x)$

⇩

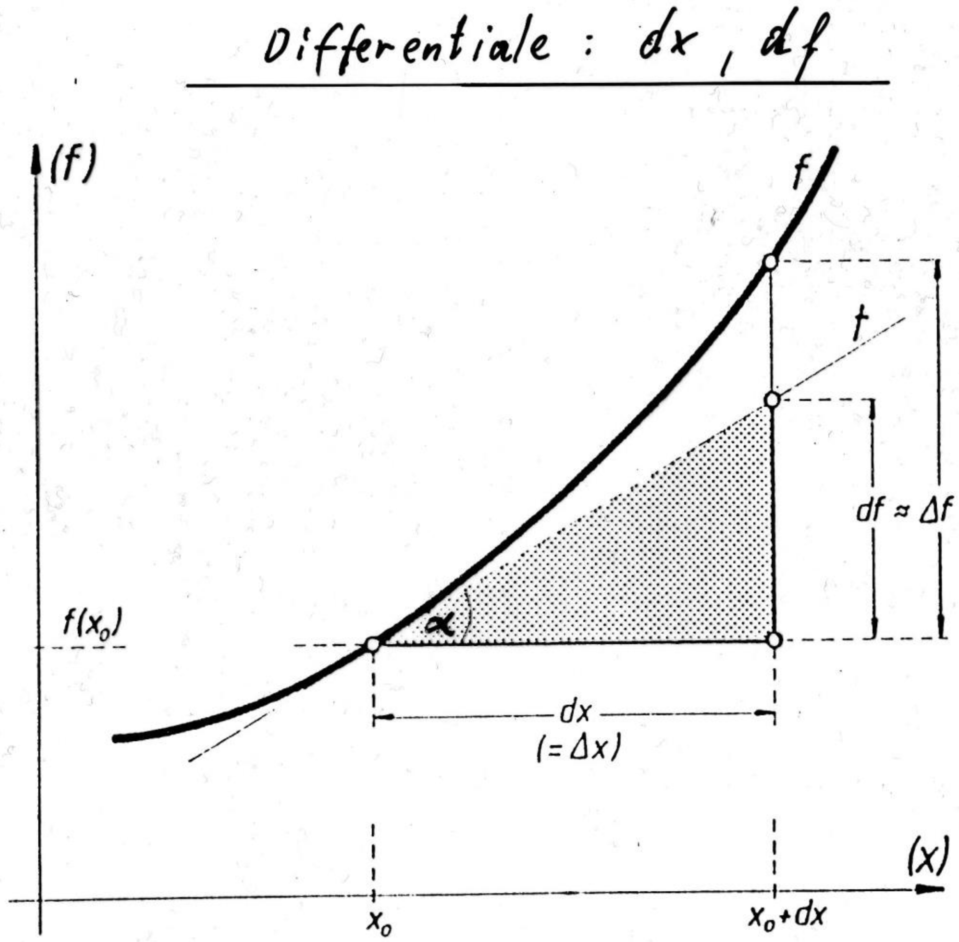
Derivation of „combined“ functions

Higher derivations:

Derivations of the first derivation **$f'(x) \Rightarrow f''(x)$** second derivation

Third derivation : **$f'''(x)$**

n^{th} derivation : **$f^{(n)}(x)$**



Application of derivative of functions:

a) Investigations of extreme values

local maximum/minimum at the point $\mathbf{x_0}$

$(f'(x_0) = 0 \quad \text{and} \quad f''(x_0) \neq 0)$

b) Curve sketching

$y = f(x)$ → Characteristics → graph of the function

c) Economics and natural sciences

Marginal function:

$P(x) = R(x) - C(x) \rightarrow \max$

Profit is defined as revenue minus costs

Necessary condition

$$P'(x) = R'(x) - C'(x) = 0$$

bzw. $R'(x) = C'(x)$

Marginal revenue is equal to marginal costs

Elasticity:

Price elasticity of demand (If the actual price changes by 1 %, by what percentage does the demand change?)

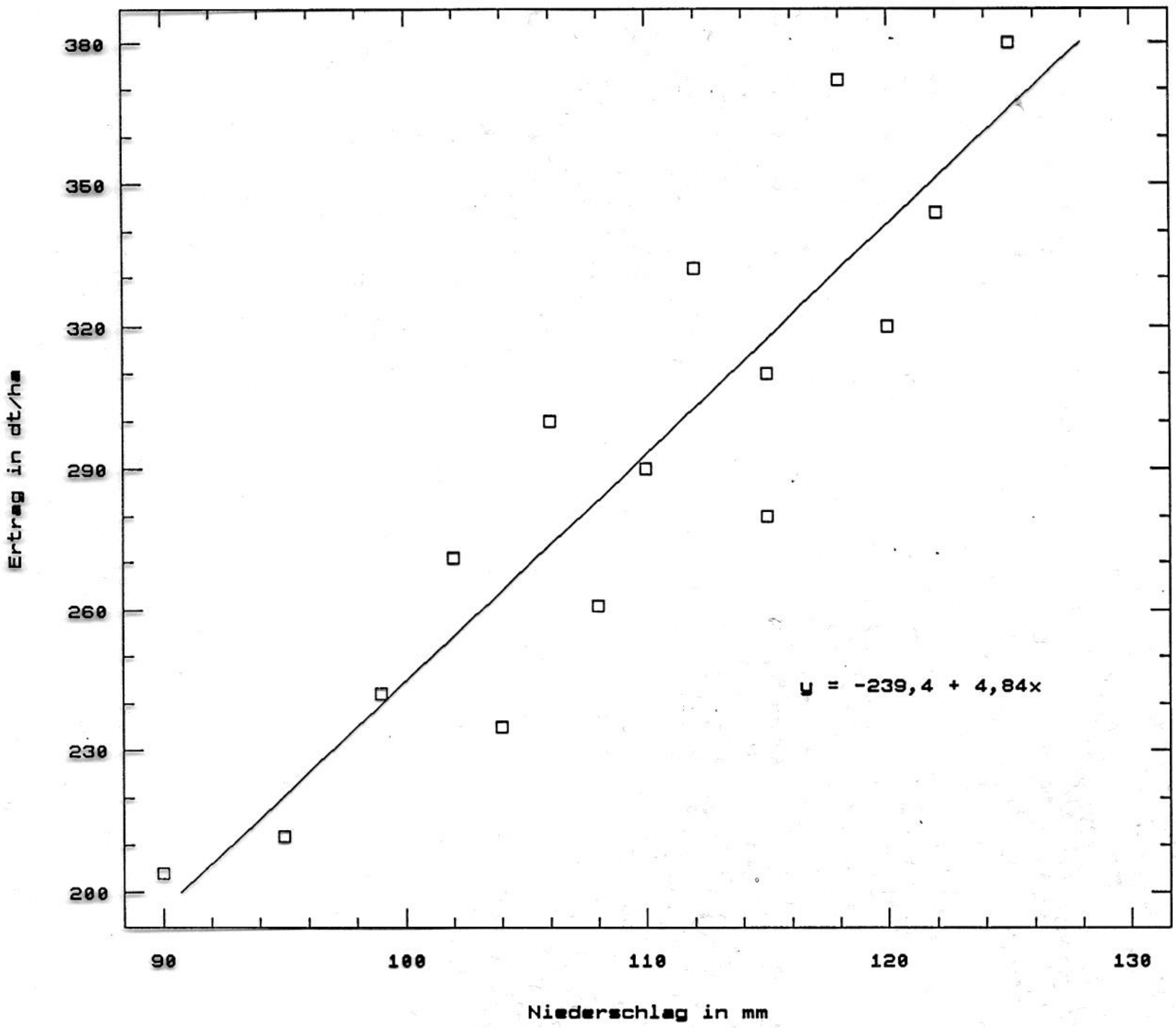
$$\varepsilon_{d,p} := d'(p) \cdot \frac{p}{d(p)}$$

Growth functions:

exponential; with saturation (logistical function,

sigmoid function (s-shape))

Analysis of trend and regression



Integral calculus

(1) Indefinite integral

Reverse of differentiation (more difficult)

$$\int f(x)\,dx = \{F(x) \mid F'(x) = f(x)\}$$

Set of all antiderivatives F of f (additive constant c)

(2) Definite integral (Riemann's Integral) defined as limit value

$$\int_a^b f(x)\,dx\,,$$

measures the area between the graph of f and the x-axis in the range of a to b.

(3) Calculation by using an antiderivative

$$\int_a^b f(x)\,dx = F(b) - F(a) =: F(x)\Big|_a^b$$

Example: $\int_1^2 3x^2\,dx = x^3\Big|_1^2 = 2^3 - 1^3 = 8 - 1 = 7$