2.1 Functions of a real variable

Definition: Be $\mathbf{M_1}$ and $\mathbf{M_2}$ two sets of real numbers.
The assignment instruction f, which allocates any $\mathbf{x} \in \mathbf{M}_1$ exactly one element $\mathbf{y} \in \mathbf{M_2}$, is called a function.

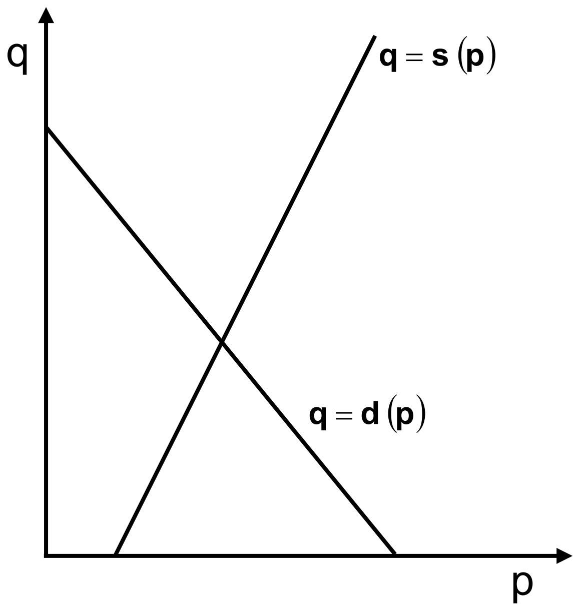
$\mathbf{f} = \left\{ (\mathbf{x},\mathbf{y}) \in \mathbf{R^2} \;\middle|\; \mathbf{x} \in \mathbf{M_1},\, \mathbf{y} = \mathbf{f}(\mathbf{x}) \in \mathbf{M_2} \right\}$ as definite mapping.

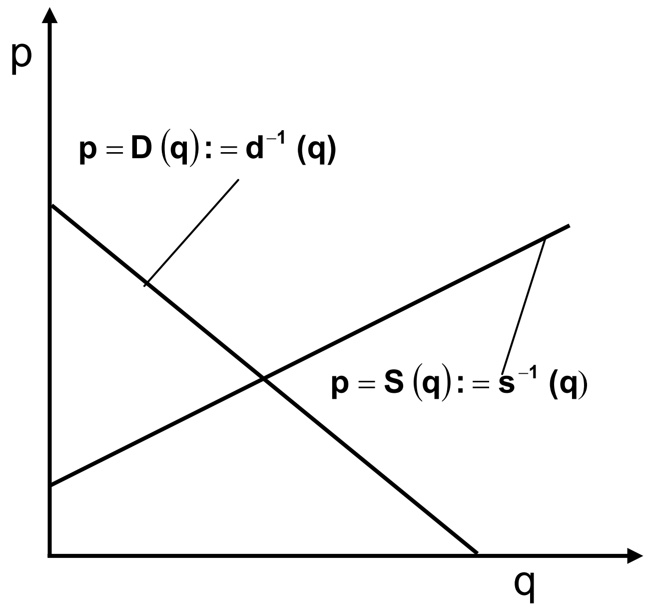
Symbolism: $\mathbf{y = f\,(x)}$ respectively $\mathbf{f : M_1 \rightarrow M_2}$

Forms of presentation:
(a) value table
(b) graphical presentation (graph)
(c) functional equality $\mathbf{y = f\,(x)}$

Example 1: Supply- and demand function, production and consumption of a good (e.g. wheat) in a country; Quantity supplied and demanded depending on the price: q = s (p)

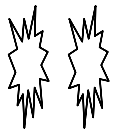
q = d (p)





Mathematical presentation Economical presentation

Example 2: growth of bacteria

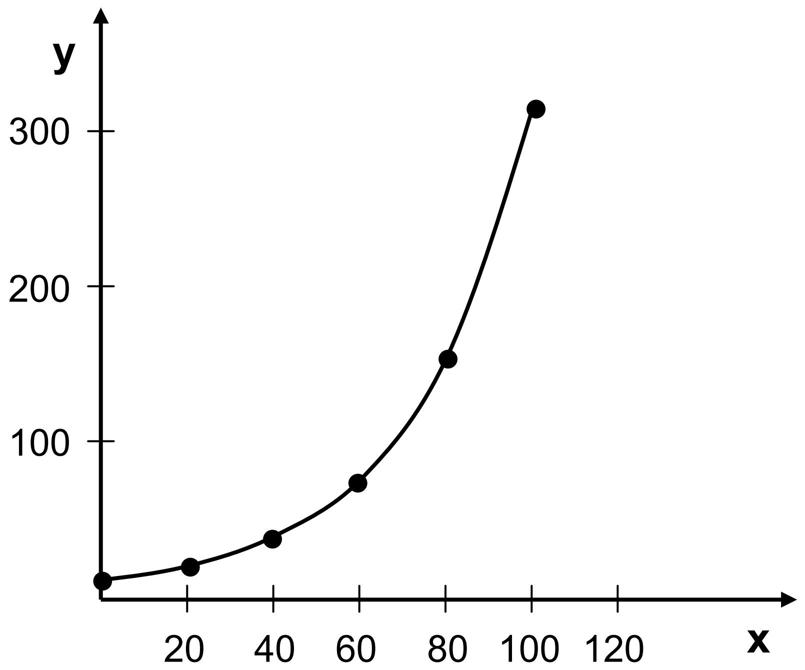
 $\rightarrow$ Division after 20 minutes $\rightarrow$ 

A nutritive substrate contains 10 bacteria in the beginning.
How many bacterias does the nutritive substrate contain 20, 40, 60 … minutes later?
How long does the development proceed in this regularity?

(a)

x	0	20	40	60	80	100	120	....
f (x)	10	20	40	80	160	320	640	

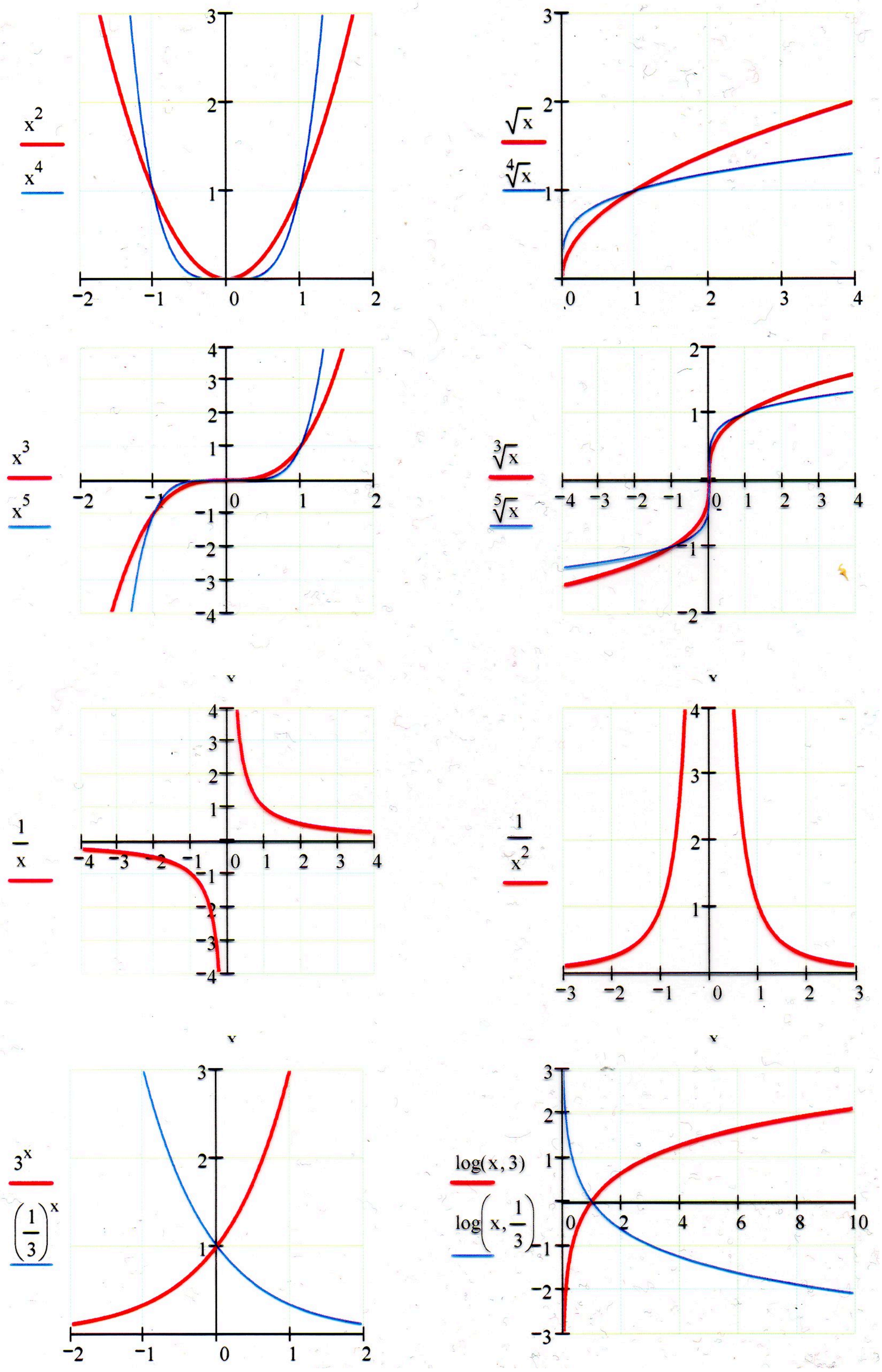
(b)



(c) $\mathbf{y = 10 \cdot 2^{\left(\frac{x}{20}\right)}}$

Graphs of basic functions y = f(x)

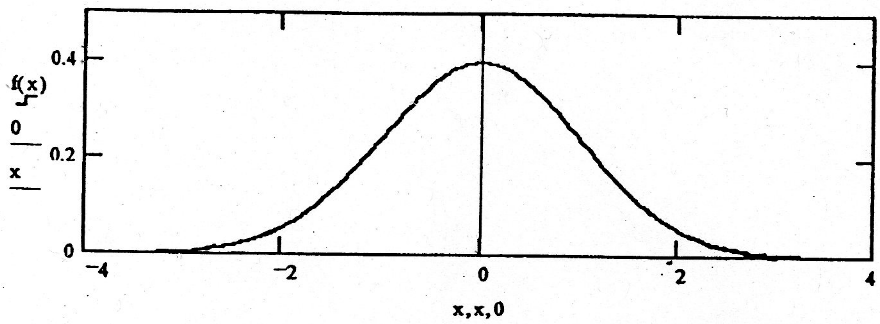
Darstellung der Grundfunktionen y = f(x)



Gauss function

Gauß'sche Glockenkurve $f(x) := \frac{1}{\sqrt{2\cdot\pi}}\cdot e^{-\frac{x^2}{2}}$

$x := -4, -3.99 .. 4$

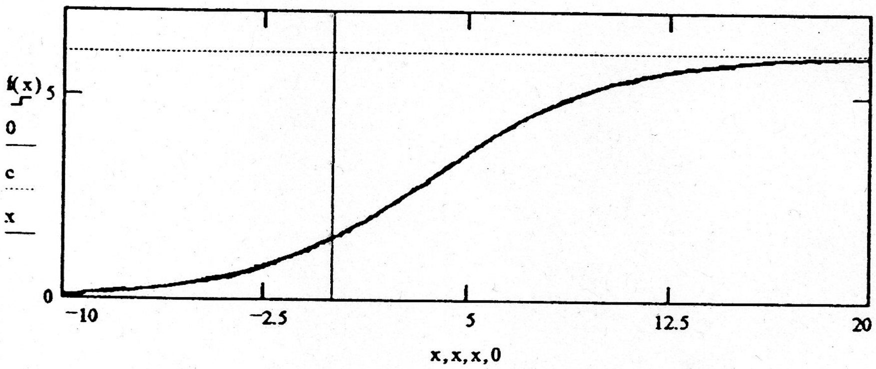


Logistic function

logistische Funktion $f(x) := \frac{c}{1 + a\cdot e^{-b\cdot x}}$

$a := 3$ $b := 0.3$ $c := 6$

$x := -10, -9.9 .. 20$



Mitscherlich function

Mitscherlich-Funktion $f(x) := c\cdot\left(1 - e^{-a-b\cdot x}\right)$

$a := 3$ $b := 0.3$ $c := 6$

$x := -20, -19.9 .. 10$