1.2 The sigma-notation

In reference to the fodder-mix-problem:

indexed variable:

x_i - amount of fodder type F_i

p_i - Price per quantity unit ($p_1 = 60,\ p_2 = 45,\ p_3 = 36$)

Solution: $x_1 = 0,\ x_2 = 8,\ x_3 = 56$

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OF: $60x_1 + 45x_2 + 36x_3 = p_1x_1 + p_2x_2 + p_3x_3 = \sum_{i=1}^{3} p_ix_i$

in general: $\sum_{i=k}^{n} a_i = a_k + a_{k+1} + \ldots + a_n$

Summation variable a_i

(A term, which mostly includes i, but not necessarily.)

Index of summation: i

lower and upper bound of summation: k, n

Examples: $\sum_{i=1}^{3} i^3 = 1^3 + 2^3 + 3^3 = 1 + 8 + 27 = 36$

$$\sum_{i=-2}^{4} 3 = 3 + 3 + 3 + 3 + 3 + 3 + 3 = 7 \cdot 3 = 21$$

Double sums, multiple sums

It is possible that the summation variable itself is a sum.

Example:

$$\sum_{i=1}^{2}\left(\sum_{j=-1}^{1} 2^i \cdot (3j+1)\right) = \sum_{j=-1}^{1} 2^1\ (3j+1) + \sum_{j=-1}^{1} 2^2\ (3j+1)$$

$$= 2\,(-3+1) \quad + \; 2\,(3\cdot 0+1) + 2\,(3+1)$$
$$+ 2^2(-3+1) + 4\,(3\cdot\, 0+1) + 4\,(3+1)$$

$$= -4 \qquad + \qquad 2 \qquad + \quad 8$$
$$-8 \qquad + \qquad 4 \qquad + \quad 16 = 18$$

$$= \sum_{j=-1}^{1} \; \sum_{i=1}^{2} 2^i \cdot (3j+1)$$

The product sign

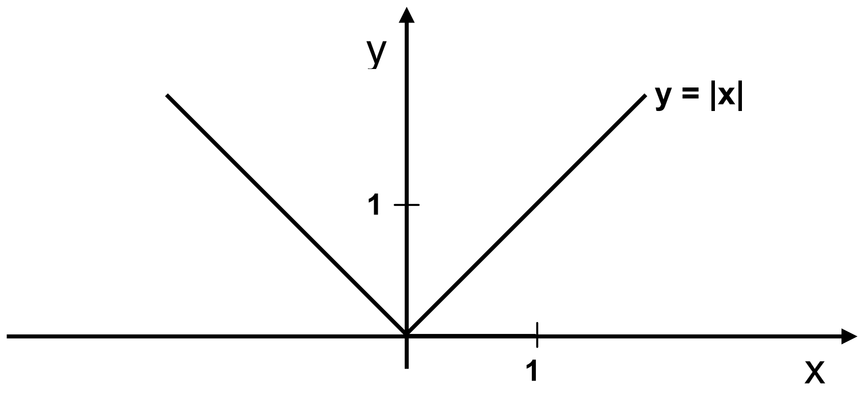
$\prod$ (Is used analogically)

Example:

$$n\,! := \; 1\cdot 2\cdot 3\cdot \;...\; \cdot n = \prod_{i=1}^{n} i$$

1.3 The absolute value of a real number

Definition: $|\mathbf{x}| := \begin{cases} \mathbf{x}, & \text{if } \mathbf{x} \geq 0 \\ -\mathbf{x}, & \text{if } \mathbf{x} < 0 \end{cases}$ **absolute value** of $\mathbf{x} \in \mathbf{R}$



Conclusions:

- $|\mathbf{x}| = \max \{\mathbf{x}, -\mathbf{x}\}$
- $\mathbf{x} \leq |\mathbf{x}|, \quad -\mathbf{x} \leq |\mathbf{x}|, \quad |-\mathbf{x}| = |\mathbf{x}|$
- Triangle inequality: $\mathbf{x}, \mathbf{y} \in \mathbf{R}$

  $|\mathbf{x} + \mathbf{y}| \leq |\mathbf{x}| + |\mathbf{y}|$

  Generalization:

  $|\mathbf{x}_1 + \ldots + \mathbf{x}_n| \leq |\mathbf{x}_1| + \ldots + |\mathbf{x}_n|$ (principle of complete induction)

Inequalities with absolute values

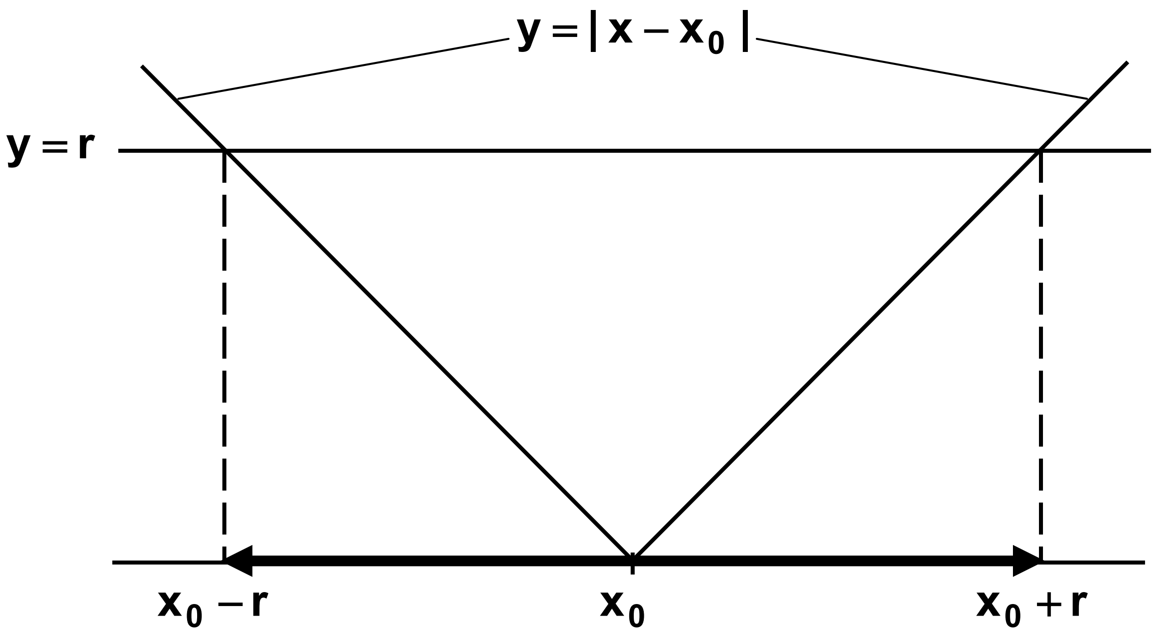
(1) $|x| \leq a \leftrightarrow -a \leq x \leq a \leftrightarrow x \in [-a, a]$

closed interval

(2) $r > 0, \quad \{x \in R \mid |x - x_0| < r\}$

$$= \{x \in R \mid x \geq x_0, x < x_0 + r\} \cup \{x \in R \mid x < x_0, x > x_0 - r\}$$
$$= \{x \in R \mid -r < x - x_0 < r\} = \{x \in R \mid x_0 - r < x < x_0 + r\} = (x_0 - r, x_0 + r)$$

open interval



1.4 Limit of a function

Fundamental idea in analysis

Use: Derivative of a function; elasticity

$$\lim_{x \to a} f(x) = b$$

means: f (x) is arbitrarily close to b for every x, which is close enough to a.

What means „close to“? $|x - a|$, $|f(x) - b|$ → small,

so

$$\lim_{x \to a} f(x) = b \text{ iff. } \forall \varepsilon > 0 \quad \exists \delta > 0 : |f(x) - b| < \varepsilon \text{ if } |x - a| < \delta .$$

Examples:

$$\lim_{x \to 3} 2x = 6,$$

$\lim_{x \to 0} \frac{1}{x}$ does not exist ($\frac{1}{x}$ is a high positive or a high negative number)