

Using Extreme Value Theory to Estimate Value-at-Risk

Martin Odening and Jan Hinrichs*

Abstract:

This article examines problems that may occur when conventional Value-at-Risk (VaR) estimators are used to quantify market risks in an agricultural context. For example, standard VaR methods, such as variance-covariance method or historical simulation, can fail when the return distribution is fat tailed. This problem is aggravated when long-term VaR forecasts are desired. Extreme Value Theory (EVT) is proposed to overcome these problems. The application of EVT is illustrated by an example from the German hog market. It turns out that multi-period VaR forecasts derived by EVT deviate considerably from standard forecasts. We conclude that EVT is an useful complement to traditional VaR methods.

Keywords: Value-at-Risk, Extreme Value Theory, Risk in Hog Production

1 Introduction

Market risk is a dominant source of income fluctuations in agriculture all over the world. This calls for indicators showing the risk exposure of farms and the effect of risk reducing measures. A concept discussed in this context is Value-at-Risk (VaR). VaR has been established as a standard tool among financial institutions to depict the downside risk of a market portfolio. It measures the maximum loss of the portfolio value that will occur over some period at some specific confidence level due to risky market factors (Jorion, 1997). Though VaR has been primarily designed for the needs in financial institutions, Boehlje and Lins (1998) and Gloy and Baker (2001) allude to its potential for applications in agribusiness. As well, Manfredo and Leuthold (2001) calculate VaR measures in order to quantify the market risk of cattle feeders. However, some well known problems have to be overcome when utilizing VaR. First, the time horizon in agricultural applications will in general be longer than in financial applications. Thus, the question arises how to extrapolate a short-term (e.g. week-to-week) volatility forecast. The usual way to achieve this is to use the square-root-rule. Unfortunately this method presumes

* Professor of Farm Management and PhD candidate, respectively. Department of Agricultural Economics, Humboldt University Berlin, Germany.

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independently, identically distributed (iid) returns and little is known about its properties if returns are not independent (for instance if they follow a GARCH process or a mean reverting process). Secondly, common VaR models have difficulties in estimating the left tail of the return distribution in particular if long time series of historical prices are not available. However, the prediction of extreme events may be of particular interest to decision makers. For example, the recent crisis of the livestock sector in the European Union, due to BSE and foot-and-mouth disease, increased the awareness of market participants towards rare market situations. In order to improve the estimation of such extreme events Diebold, Schuerman, and Stroughair (1998) suggest the use of Extreme Value Theory (EVT). EVT can be considered as a state-of-the-art procedure for estimating the downside risk of a distribution. However, to the knowledge of the authors it has not been applied in the agribusiness yet.

The objective of this paper is threefold. First, we intend to pinpoint the aforementioned pitfalls of traditional VaR models. Second, we want to explore the applicability of EVT for practical agricultural problems. The German hog market serves as an illustrative example for this. Finally, a comparison of the performance between EVT and standard VaR techniques shall support the decision whether more or less sophisticated methods are appropriate in order to assess market risks in agriculture.

The paper is organized as follows: Section 2 briefly describes the VaR concept and some related problems. Section 3 introduces EVT and shows how this concept can be used to estimate the tails of return distributions. In Section 4 we apply EVT to the German hog market and derive 12-week VaR forecasts for the returns in feeder pig production and hog finishing. The results are compared with the outcome of standard procedures, namely historical simulation and the variance-covariance method. The paper ends with a discussion of the strengths and weaknesses of VaR and EVT.

2 Value-at-Risk

2.1 Definition

Briefly stated, VaR measures the maximum expected loss over a given time period at a given confidence level that may arise from uncertain market factors. Call W the value of an asset or a

portfolio of assets and $V = W_{t_1} - W_{t_0}$ the random change (revenue) of this value during the period

$h = \mathbf{D}t = t_1 - t_0$, then VaR is defined as follows:

$$\text{VaR} = E(V) - V^* . \quad (1)$$

$E(V)$ is the expectation of V and the critical revenue V^* is defined by:

$$\int_{-\infty}^{V^*} f(v)dv = \text{Prob}(v \leq V^*) = p . \quad (2)$$

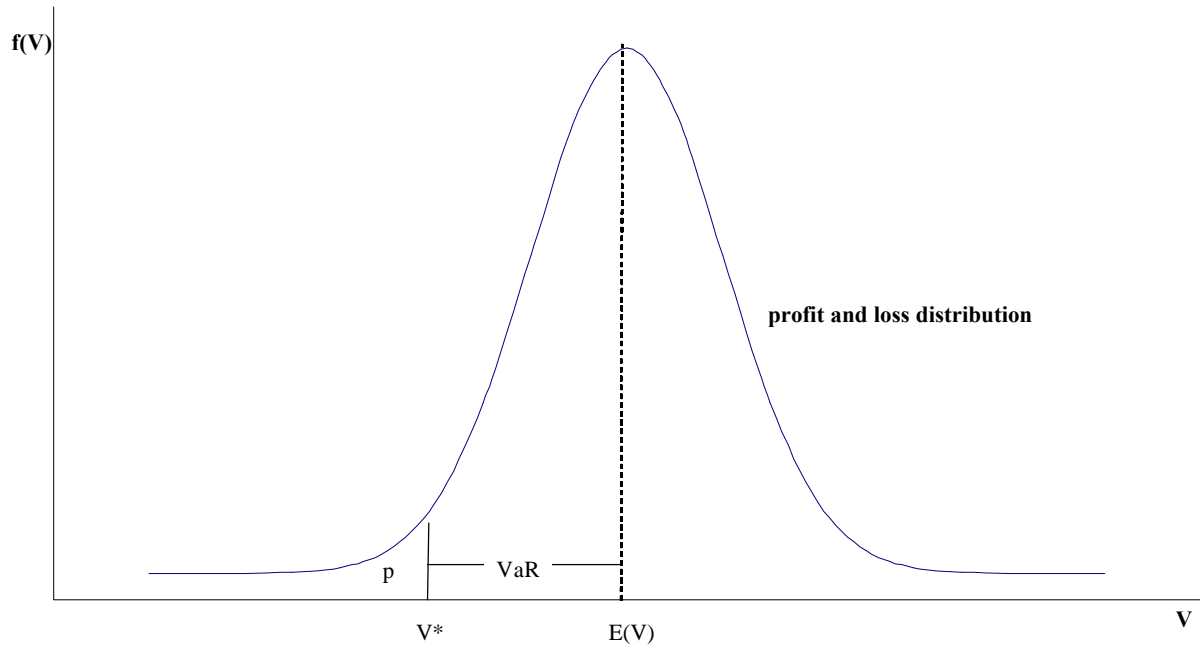
Using the identity $V = W_{t_0} \cdot X$ with $X = \ln(W_{t_1}/W_{t_0})$ VaR can also be expressed in terms of the critical return X^* :

$$\text{VaR} = W_{t_0} (E(X) - X^*) \quad (3)$$

where $E(X)$ and X^* are defined analogous to $E(V)$ and V^* .

Figure 1 illustrates the concept graphically. From equation (2) and figure 1, it is obvious that the calculation of VaR boils down to finding the p -quantile of the random variable V (i.e. the profit-and-loss distribution). An advantage of VaR is that it can be intuitively understood. Standard risk criteria like stochastic dominance or certainty equivalents rely on the entire distribution. Opposed to that VaR considers just the left tail of the distribution. That means risk is viewed as bad outcome. Moreover VaR is easy to calculate and it is not difficult to include multiple uncertain market factors (e.g. commodity prices, futures prices or interest rates) into the analysis. However, the implication of this indicator for risk management is not straightforward. The choice of the confidence level is somewhat arbitrary and in general, consistency with the expected utility theory is not guaranteed. The reason is that VaR only quantifies the probability that a loss exceeds a certain level, but the magnitude of such a loss is not specified (Harlow, 1991). Despite this deficiency, our basic premise in this paper is that VaR contains useful information about the (downside) risk exposure of an enterprise and we focus on the question *how* to determine this risk indicator. Alternative methods to calculate VaR are briefly summarized in the next section.

Figure 1: Definition of VaR



2.2 Methods of VaR Calculation

The literature offers three standard procedures for VaR estimation, namely the variance-covariance method (VCM), Monte Carlo simulation (MS) and historical simulation (HS), all showing specific advantages and disadvantages. A detailed treatment of these methods can be found in (Jorion, 1997) and (Dowd, 1998). Manfredo and Leuthold (1999) discuss the pros and cons of these estimation procedures. This paper provides a brief overview and pinpoints some deficiencies that we try to overcome later.

Variance-Covariance Method

The VCM (also called parametric approach or delta-normal method) determines VaR directly as a function of the volatility of the portfolio return \mathbf{s} . If normality of the returns is assumed, VaR can be determined as:

$$\text{VaR} = W_{t_0} \cdot c \cdot \mathbf{s} \cdot \sqrt{h} . \quad (4)$$

Herein c denotes the p -quantile of the standard normal distribution. \sqrt{h} is a scaling factor that adapts the time horizon of the volatility to the length of the holding period h . The problems that may arise when using such a scaling factor are discussed in section 2.4. In the case of a portfolio that consists of n assets, the volatility of the portfolio return is calculated according to:

$$\mathbf{s}_p = \left(\sum_{i=1}^n \sum_{j=1}^n w_i \cdot w_j \cdot \mathbf{s}_{ij} \right)^{0.5} \quad (5)$$

where w_i and w_j are the weights of assets i and j and \mathbf{s}_{ij} is the covariance of their returns.

An apparent advantage of the VCM is its ease of computation. If the normality assumption holds, VaR figures can be simply translated across different holding periods and confidence levels. Moreover, time-varying volatility measures can be incorporated and what-if-analyses are easy to conduct. On the other hand, the normality assumption is frequently criticized. There is empirical evidence that return distributions are fat tailed, and in that case the VCM will underestimate the VaR for high confidence levels. Further problems occur if the portfolio return depends in a nonlinear way on the underlying risk factors, which is typically the case when options are included in the portfolio.

Monte Carlo Simulation

With this method the entire distribution of the value change of the portfolio is generated, and VaR is measured as an appropriate quantile from this relative frequency distribution. The simulation involves the following steps:

Selection of distributions for the changes of the relevant market factors (e.g. commodity prices) and estimation of the appropriate parameters, in particular variances and correlations,

simulation of random paths for the market factors,

evaluation of the portfolio for the desired forecast horizon ("mark-to-market"),

calculation of the gains or losses related to the current portfolio value,

repetition of the three aforementioned steps until a sufficient accuracy is gained, and,

ordering of the value changes in ascending order and determination of the frequency distribution.

The main advantage of the Monte Carlo simulation is the ability to handle different return distributions. Harmful are the high costs of computation in the case of complex portfolios.

Historical Simulation

Historical simulation (HS) resembles the Monte Carlo simulation regarding the iteration steps. The difference is that the value changes of the portfolio are not simulated by means of a random number generator, but directly calculated from observed historical data. That means, VaR estimates are derived from the empirical profit-and-loss distribution. Hence, no explicit assumption about the return distribution is required. However, this procedure implicitly assumes a constant (stable) distribution of the market factors. A general problem arises from the fact that the empirical distribution function, while being relatively smooth around the mean, shows discrete jumps in the tails due to the small number of extreme sample values. The higher the desired confidence level, the more uncertain the estimation of the corresponding quantile becomes. Accordingly, VaR estimates based on HS are sensitive to changes in the data sample. In addition, it is not possible to predict events that are worse than the maximal loss during the sample period. The extreme value theory, described in section 3, offers an opportunity to avoid these problems.

2.3 Modeling the Return Distribution

If a parametric approach to VaR estimation is utilized, the question arises which distribution function fits best to the observed changes of the market factors. As mentioned above, it is widely recognized in the literature that empirical return distributions of financial assets are characterized by fat tails. Wei and Leuthold (1998) provide some evidence that agricultural price series may also exhibit fat tails. With respect to modeling the underlying stochastic process, two consequences can be deduced (Jorion, 1997, p. 166 f.): either one uses a leptokurtic distribution, e.g. a t -distribution, or one resorts to a model with stochastic volatility. Of course both approaches can be combined. The observation of volatility clusters in high frequency (i.e. daily) data favors the use of models with stochastic volatilities. The changing of phases of relatively small and relatively high fluctuations of returns can be captured with GARCH models. Yang and Brorsen (1992) demonstrate that GARCH models are not only relevant for financial applications, but also appropriate to describe the development of daily spot market prices of agricultural commodities. Herein a stochastic process of the form

$$X_t = \mathbf{m}_t + \mathbf{s}_t \mathbf{e}_t \quad (6)$$

is assumed for the returns, where \mathbf{e}_t are iid random variables. In most applications normal or t -distributions for the disturbance variable \mathbf{e}_t are presumed. In a GARCH(1,1) process, the variance \mathbf{s}_t^2 develops according to

$$\mathbf{s}_{t+1}^2 = \mathbf{w}^2 + \mathbf{d}X_t^2 + \mathbf{b}\mathbf{s}_t^2 \quad (7)$$

with $\mathbf{w} = \mathbf{g}\bar{\mathbf{s}}^2 > 0$, $\mathbf{d} \geq 0$, $\mathbf{b} \geq 0$, $\mathbf{d} + \mathbf{b} < 1$

$\bar{\mathbf{s}}^2$ is a long-term average value of the variance, from which the current variance can deviate in accordance with (7). Obviously the use of models with stochastic volatility implies a permanent updating of the variances and thus the VaR forecasts.

While conditional models are superior for short-term forecasts, their value vanishes with increasing time horizon. Christoffersen and Diebold (2000) argue that the recent history of data series has little to tell about the probability of events occurring far in the future. This applies especially to the prediction of rare events like disasters, which are assumed to be stochastically independent. Therefore Danielsson and de Vries (2000) recommend to derive predictions about extreme events from unconditional distributions.

2.4 Long-term Value-at-Risk

Much of this paper is motivated by the supposition that the relevant VaR horizon in agricultural applications in general will be longer than in a financial context, where one-day or few-day forecasts dominate. In general the desired forecast horizon and the observation frequency of the data will deviate. One may think of a farmer who wishes to determine the VaR for a six-months-period according to his production cycle having weekly price data at hand. Basically two methods exist to calculate long-term VaRs: either one measures the value changes that occur during the entire holding period, that means, the VaR is estimated on the basis of six month's returns. Alternatively, a short-term VaR can be extrapolated to the desired holding period (time scaling). The first procedure is applicable independent of the return distribution. However, it has the serious drawback that the number of observations is strongly reduced. If, for instance, weekly data are available over a period of 10 years, a six-month VaR is based on 20 observations only, since the measurement periods should be non-overlapping. The second method avoids this problem. In practice the time-scaling is conducted by means of the square-root-rule

$$\text{VaR}(h) = \text{VaR}(1) \cdot \sqrt{h}. \quad (8)$$

$\text{VaR}(1)$ and $\text{VaR}(h)$ denote the one-period VaR and the h -period VaR, respectively. Diebold et al. (1997) point out that the correctness of the square-root-rule relies on three conditions. First, the structure of the considered portfolio may not change in the course of time. Secondly, the returns must be identically and independently distributed, and thirdly, they must be normally distributed. Section 3.1 discusses the consequences of non-normality for time aggregation. At this point, we ask what would happen if the iid assumption is not fulfilled. Though a general answer to this question is not available, Drost and Nijman (1993) provide a formula for the correct time aggregation of a GARCH process. For the GARCH(1,1) process described above, the h -period volatilities can be determined from the one-period volatilities as follows¹:

$$\mathbf{s}_{i+1}^2(h) = \mathbf{w}(h)^2 + \mathbf{d}(h)X_i^2(h) + \mathbf{b}(h)\mathbf{s}_i^2(h) \quad (9)$$

$$\text{with } \mathbf{w}(h) = h\mathbf{w} \frac{1 - (\mathbf{d} + \mathbf{b})^h}{1 - (\mathbf{d} + \mathbf{b})}$$

$$\mathbf{d}(h) = (\mathbf{d} + \mathbf{b})^h - \mathbf{b}(h)$$

and $|\mathbf{b}(h)| < 1$ as solution of the quadratic equation

$$\frac{\mathbf{b}(h)}{1 + \mathbf{b}^2(h)} = \frac{a - (\mathbf{d} + \mathbf{b})^h - b}{a(1 + (\mathbf{d} + \mathbf{b})^{2h}) - 2b}$$

The coefficients a and b are defined as:

$$a = h(1 - \mathbf{b})^2 + 2h(h-1) \frac{(1 - \mathbf{d} - \mathbf{b})^2(1 - 2\mathbf{d}\mathbf{b} - \mathbf{b}^2)}{(\mathbf{k} - 1)(1 - (\mathbf{d} + \mathbf{b})^2)} \\ + 4 \frac{(h-1 - h(\mathbf{d} + \mathbf{b}) + (\mathbf{d} + \mathbf{b})^h)(\mathbf{d} - \mathbf{d}\mathbf{b}(\mathbf{d} + \mathbf{b}))}{1 - (\mathbf{d} + \mathbf{b})^2}$$

$$b = (\mathbf{d} - \mathbf{d}\mathbf{b}(\mathbf{d} + \mathbf{b})) \frac{1 - (\mathbf{d} + \mathbf{b})^{2h}}{1 - (\mathbf{d} + \mathbf{b})^2}$$

where \mathbf{k} denotes the kurtosis of the return distribution.

Comparing (9) with (8) reveals systematic differences, which become larger with increasing h . If h goes to infinity, \mathbf{d} and \mathbf{b} in (9) converge to zero and hence the stochastic terms vanish, whereas the first deterministic term increases. That means that the average levels of the h -period volatility coincide in both cases, but the square-root-rule magnifies the fluctuations of the volatility, while

they actually become smaller with increasing time horizon. Diebold et al. (1997) illustrate the magnitude of the difference of both methods of volatility forecasting by means of simulation experiments. Similar calculations in the context of our application are presented in section 4.

3 Extreme Value Theory

From the discussion in section 2.2, some pitfalls of traditional methods of VaR estimation became obvious, in particular if the prediction of very rare events is desired and leptokurtic distributions are involved. Now we turn to the Extreme Value Theory (EVT) in order to improve the estimation of extreme quantiles². EVT provides statistical tools to estimate the tails of probability distributions. Some basic concepts are briefly addressed below. A much more comprehensive treatment can be found in Embrechts, Klüppelberg, and Mikosch (1997).

3.1 Basic concepts

A main objective of the EVT is to make inferences about sample extrema (maxima or minima)³. In this context the so called Generalized Extreme Value distribution (GEV) plays a central role. Using the Fisher Tippet theorem, it can be shown that for a broad class of distributions the normalized sample maxima (i.e. the highest values in a sequence of iid random variables) converge towards the Generalized Extreme Value distribution with increasing sample size. If X_1, X_2, \dots, X_n are iid random variables from an unknown distribution F , and let a_n and b_n be appropriate normalization coefficients, then for the sample maxima $M_n = \max(X_1, X_2, \dots, X_n)$ holds:

$$p \lim \left(\frac{M_n - b_n}{a_n} \leq x \right) = H(x) \quad (10)$$

where $p \lim$ means the limit of a probability for $n \rightarrow \infty$ and $H(x)$ denotes the GEV, which is defined as follows:

¹ Kroner, Kneafsey, and Claessens (1995) provide an alternative, recursive formula for the time aggregation of a GARCH(1,1) process, which is somewhat simpler than the Drost-Nijman-formula. However, we prefer the latter, since it allows to incorporate the kurtosis of the distribution of the random shocks.

² We emphasize that EVT is not the only method to cope with extreme events and fat tailedness. For example, Li (1999) uses a semiparametric approach to VaR estimation, which takes into account skewness and kurtosis of the return distribution in addition to the variance. Moreover, stress testing is a rather widespread technique that may be used as a complement to traditional VaR methods. It gauges the vulnerability of a portfolio under extreme hypothetical scenarios.

³ Embrechts, Klüppelberg, and Mikosch (1997, p. 364) express the objective of EVT vividly as “mission improbable: how to predict the unpredictable”.

$$H(x) = \begin{cases} \exp\left(- (1 + \mathbf{x}x)^{-1/\mathbf{x}}\right) & \text{if } \mathbf{x} \neq 0 \\ \exp(-e^x) & \text{if } \mathbf{x} = 0 \end{cases} \quad (11)$$

The GEV includes three extreme value distributions as special cases, the Frechet distribution ($\mathbf{x} > 0$), the Weibull distribution ($\mathbf{x} < 0$), and the Gumbel distribution ($\mathbf{x} = 0$). Depending on the parameter \mathbf{x} , a distribution F is classified as fat tailed ($\mathbf{x} > 0$), thin tailed ($\mathbf{x} = 0$), and short tailed ($\mathbf{x} < 0$). In the present context the focus is on the first class of distributions, which includes for example the t-distribution and the Pareto distribution, but not the normal distribution. Embrechts, Klüppelberg, and Mikosch (1997, p. 131) prove that the sample maxima of a distribution exhibiting fat tails converges towards the Frechet distribution $F(x) = \exp(x^{\mathbf{a}})$, if the following condition is satisfied:

$$1 - F(x) = x^{-1/\mathbf{a}} L(x). \quad (12)$$

Equation (12) requires that the tails of the distribution F behave like a power function. $L(x)$ is a slowly varying function and $\mathbf{a} = 1/\mathbf{x}$ is the tail index of the distribution. The smaller \mathbf{a} is, the thicker are the tails. Moreover, (12) indicates that inferences about extreme quantiles of a possibly unknown distribution of F can be made as soon as the tail index \mathbf{a} and the function L have been determined. Section 3.2 describes an estimation procedure for \mathbf{a} .

The results of the EVT are also relevant for the aforementioned task of converting short-term VaRs into long-term VaRs. Assume $P(|X| > x) = Cx^{-\mathbf{a}}$ applies to a single-period return X for large x . Then we have for a h -period return (Danielsson and de Vries, 2000):

$$P(X_1 + X_2 + \dots + X_h > x) = hCx^{-\mathbf{a}}. \quad (13)$$

(13) holds due to the linear additivity of the tail risks of fat tailed distributions. It follows that a multi-period VaR forecast of a fat tailed return distribution under the iid assumption is given by:

$$\text{VaR}(h) = \text{VaR}(1)h^{1/\mathbf{a}}. \quad (14)$$

If the returns have finite variances then $\mathbf{a} > 2$ and thus a smaller scaling factor applies than postulated by the square-root-rule (Danielsson, Hartmann, and de Vries, 1998). Obviously the square-root-rule is not only questionable if the iid assumption is violated, but also if the return distribution is leptokurtic.

3.2 Estimation of the Tail Index

Several methods exist to estimate the tail index of a fat tailed distribution from empirical data. The most popular is the Hill estimator (Diebold, Schuerman, and Stroughair, 1998). To implement this procedure, the observed losses X are arranged in ascending order: $X_1 > X_2 > \dots > X_k > \dots > X_n$. The tail index $\mathbf{a} = 1/\mathbf{x}$ then can be estimated as follows:

$$\hat{\mathbf{a}}(k) = \left(\frac{1}{k} \sum_{i=1}^k \ln X_i - \ln X_{k+1} \right)^{-1}. \quad (15)$$

The function $L(x)$ in (12) is usually approximated by a constant C . An estimator for C is (Embrechts, Klüppelberg, and Mikosch, 1997, p. 334):

$$\hat{C}_k = \frac{k}{n} X_{k+1}^{\hat{\mathbf{a}}}. \quad (16)$$

This leads to the following estimator for the tail probabilities and the p -quantile:

$$\hat{F}(x) = p = \frac{k}{n} \left(\frac{X_{k+1}}{x} \right)^{\hat{\mathbf{a}}}, x > X_{k+1} \quad \text{and} \quad (17)$$

$$\hat{x}_p = F^{-1}(x) = X_{k+1} \left(\frac{k}{np} \right)^{\frac{1}{\hat{\mathbf{a}}}}. \quad (18)$$

It can be shown that the Hill Estimator is consistent and asymptotically normal (Diebold, Schuerman, and Stroughair, 1998).

The implementation of the estimation procedure requires to determine the threshold value X_k , i.e. the sample size k , on which the tail estimator is based. It is well known that the estimation results are strongly influenced by the choice of k . Moreover, a trade-off exists: the more data are included in the estimation of the tail index \mathbf{a} , the smaller the variance becomes. Unfortunately, the bias increases at the same time because the power function in (12) applies only to the tail of the distribution. In order to solve this problem, Danielsson et al. (2001) develop a bootstrap method for the determination of the sampling fraction k/n . The different steps of this iterative procedure are described in the appendix.

4 Application to Hog Production

4.1 Model and Data

Following Manfredo and Leuthold (2001), who investigate the market risks in US cattle feeding, we use the VaR approach to quantify the market risk in hog production under German market conditions. While the Common Agricultural Policy (CAP) dampens price fluctuations for many agricultural commodities in the EU, the hog market has proven to be very volatile. Our target is the determination of a 12-week VaR for three types of producers: 1) a specialized feeder pig producer, 2) a farmer who specializes in hog finishing and purchases feeder pigs, and 3) a farrow-to-finish operation. We assume that prices of feeder pigs and finished hogs are not fixed by forward contracts; rather feeder pigs and finished hogs are bought and sold at current spot market prices. The gross margin (cash flow) CF at time t associated with these production activities is defined as

$$CF_t = a \cdot P_t - \sum_{i=1}^K b_i Z_{it}. \quad (19)$$

Formally the gross margin can be considered as a portfolio consisting of a long-position (the product price P) and several short-positions (the factor prices Z_i). Thus (5) can be applied to this margin. The portfolio weights a and b_i now have to be interpreted as technical coefficients (slaughtering weight, fodder consumption etc.). Thereby we imply a fixed production technology, which is not unusual in risk management applications (cf. Manfredo and Leuthold (2001) or Kenyon and Clay (1997)). Empirical investigations of Odening and Musshoff (2002) indicate that the market risk in hog production in Germany is mainly caused by the prices of feeder pigs and finished hogs. Other items, e.g. fodder costs, have an impact on the level of the gross margins, but they do not contribute to the fluctuations of the cash flow. As mentioned above, prices for the most important fodder components are (still) stabilized by market intervention in the CAP framework. Therefore we do not include them in the following calculation. Due to this, the VaR calculation simplifies considerably. In what follows, we display the VaRs for the feeder pigs prices (the perspective of the specialized feeder pigs producer), for the finished hog prices (the perspective of the farrow-to-finish operation) and for the hog finishing margin (the perspective of the specialized hog producer who buys feeder pigs and sells

finished hogs). The weights of feeder pigs and finished hogs (slaughter weight) are assumed to be 20 kg and 80 kg (44 lbs and 176 lbs), respectively.

Note that in a strict sense we do not display a Value-at-Risk, but rather a Cash-Flow-at-Risk (CFaR) (Dowd, 1998, p. 239 f.)⁴. Despite the formal analogy of both concepts, one should have in mind the differences when it comes to an economic interpretation of the figures: VaR quantifies the loss of value of an asset, whereas CFaR addresses a flow of money. The knowledge of a CFaR is presumably valuable in the context of risk-oriented, medium-term financial planning. However, conclusions about the financial endangerment of the farm should be drawn carefully, since the initial cash flow level, as well as the duration of the cash flow drop, should be taken into account. Experience shows that specialized livestock farms are able to endure losses if such a period does not persist too long and appropriate profits have been earned before.

Our empirical analysis is based on time series of prices for finished hogs and feeder pigs published by the Zentrale Markt- und Preisberichtsstelle (ZMP), a German market reporting agency. The data consist of weekly price quotations which are reported by hog producers and slaughter houses in East Germany. The time series spans the period from January 1994 until October 2001, i.e. 405 observations are available. Prices are measured in Euro per kg live weight and slaughter weight for feeder pigs and finished hogs, respectively. The latter refer to an average meat quality insofar as prices for different grades are aggregated using the trading volumes as weights. Note that in the following empirical analysis price changes are used rather than the absolute prices⁵.

4.2 Empirical results

In line with the discussion in section 2.3, the first step in VaR calculation is to clarify what kind of distributions underlie the market factors, i.e. finished hog prices, feeder pig prices and the hog finishing margin. This task breaks down into two questions. First, should a conditional or an unconditional model be used, and second, are the respective distributions fat tailed or thin tailed?

⁴ Nevertheless we continue to speak of VaR (in a broader sense) below.

⁵ In financial applications it is common to analyze log returns instead of absolute changes. Their advantage is to be independent of the price level. However, problems occur if values become negative. While this is impossible for prices it may happen to the hog finishing margin. A natural way to prevent this problem is to model the risky components of the margin, i.e. the input and output prices. While VCM and HS are predestinated for an analysis of various risky market factors, EVT is not, since it is essentially a univariate approach.

To answer the first question, a Lagrange Multiplier test is employed to test the presence of conditional heteroscedasticity (Greene 2000, p. 808). This test rejects the null hypothesis of homoscedasticity for the weekly changes of finished hog prices and of the hog finishing margin, therefore, a GARCH(1,1) model for all three time series is estimated⁶. The estimated parameter values are summarized in table 1. All estimated parameters are significant. The standardized residuals $\hat{\epsilon}_t / s_t$ indicate no autocorrelations on a 1% level of significance. This applies also to the squared standardized residuals with exception of the feeder pig price series. Thus the inclusion of further lags into the GARCH model does not appear necessary. Inserting the parameters in table 1 into (7) yields 1-week volatility forecasts. Next, the 1-week volatility forecasts are projected on a 12-week horizon. This is conducted with the square-root-rule (8) and alternatively with the Drost-Nijman formula (9). The results for the volatility of the finished hog price changes are represented in figure 2. The corresponding results for the volatility forecasts of the feeder pig prices and the hog finishing margin, which are not presented here, look very similar.

Table 1: Parameters of the GARCH (1,1)-Models

parameter	price of feeder pigs	price of finished hogs	hog finishing margin
w	0.0009** (5.71)	0.0007** (3.79)	0.8626* (1.81)
d	0.7100** (6.24)	0.4439** (4.22)	0.1641** (4.24)
b	0.1728** (4.26)	0.2769** (2.34)	0.7629** (12.21)

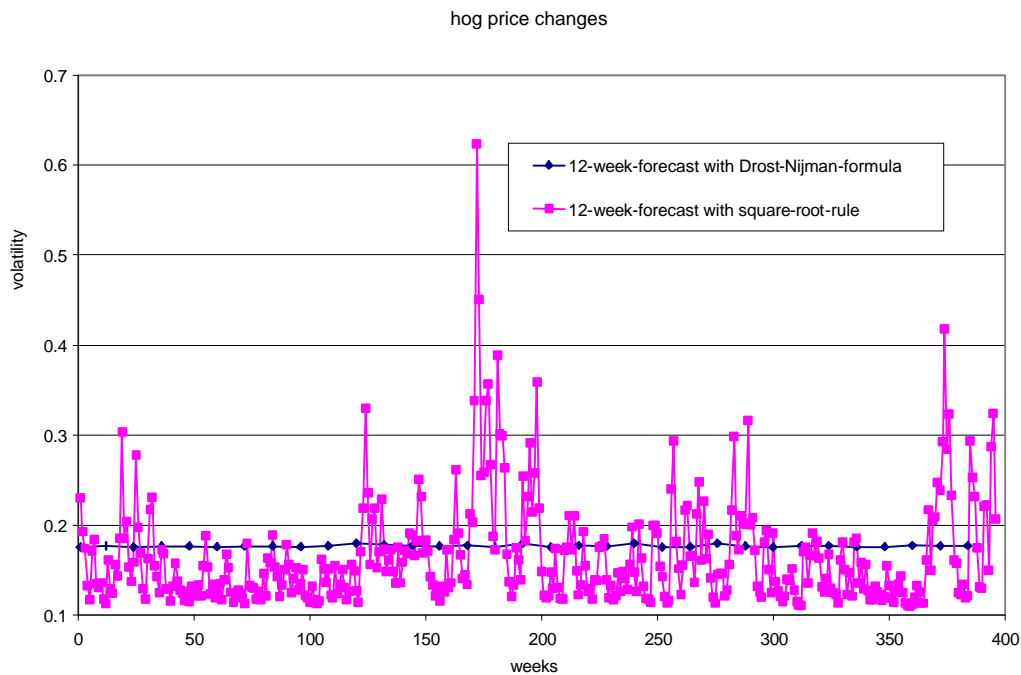
* level of significance 95% ** level of significance 99%, t-values in parentheses

Figure 2 confirms the theoretical considerations in section 2.4. The square-root-rule cannot be regarded as a suitable approximation for a correct time aggregation of the volatility in GARCH models. The actual fluctuations of the volatility are substantially smaller than shown by multiplication with the factor $\sqrt{12}$. That means that VaR forecasts, that are based on this methodology, lead to a permanent overestimation and an underestimation of the true 12-week

⁶ We refrain from estimating a Bi-GARCH-model for the feeder pig prices and pig prices to estimate the volatility and the VaR of the hog finishing margin. Instead, a univariate GARCH model for the margin is estimated. This corresponds to the procedure that is used later for the EVT application. It takes into account that EVT at its present stage is only applicable to univariate distributions.

VaRs. The correctly determined fluctuation of the 12-week volatility appears so small that – in accordance with the arguments of Danielsson and de Vries (2000) – the subsequent application of EVT is based on unconditional distributions, regardless of the measurement of conditional heteroscedasticity in weekly price changes.

Figure 2: Temporal aggregation of time varying volatilities (GARCH 1,1)

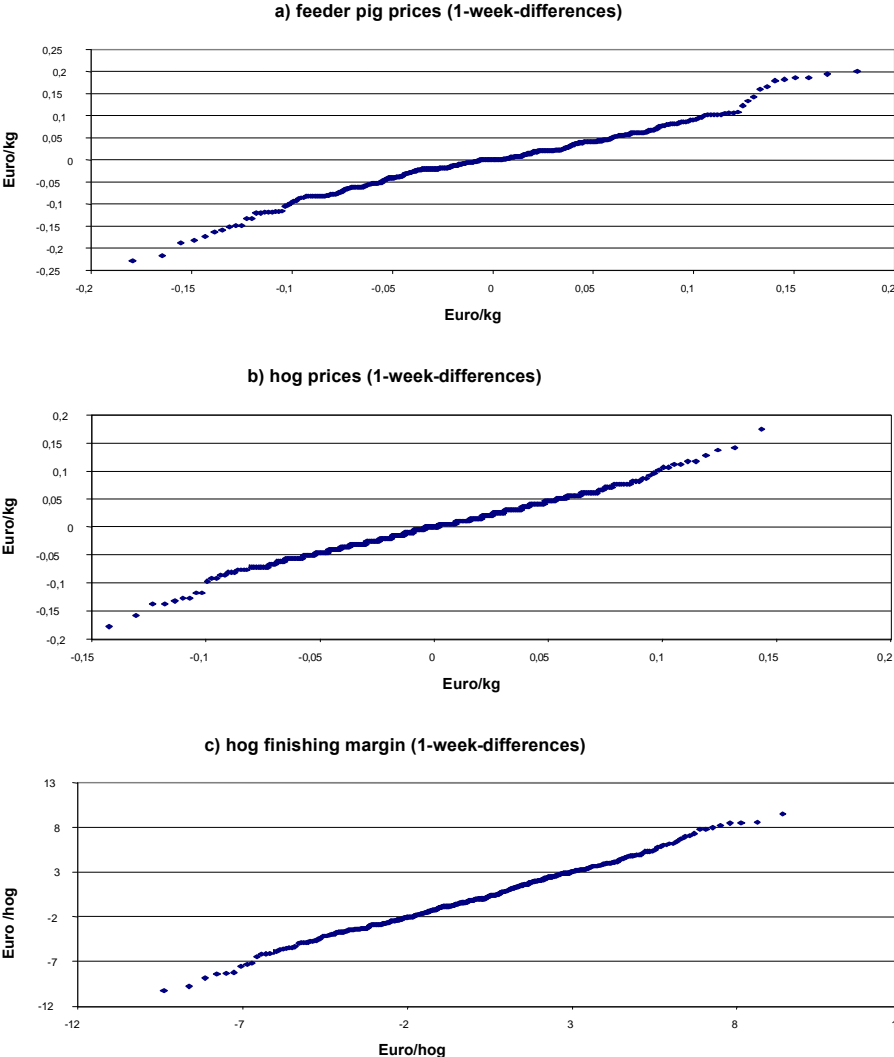


Next, we turn to the question whether the considered time series are fat tailed or not. This issue can be inspected by QQ-plots, which compare the quantiles of an empirical distribution and a theoretical reference distribution. If the data points are approximately located on a straight line, it can be assumed that the observed data follow the reference distribution. In figure 3 the normal distribution is chosen as a reference distribution.

Figure 3 indicates a positive excess for the weekly changes of feeder pig prices and finished hog prices whereas the interpretation of the QQ-plot of the hog finishing margin is less clear. The realization of a Kolmogorov-Smirnoff goodness-of-fit test supports the conjecture that the analyzed series are not normally distributed. The null hypothesis is rejected on a 5% level for all three distributions. Finally, the Jarque-Bera-test, which summarizes deviations from the normal distribution with respect to skewness and kurtosis, provides further evidence about the non-normality of the distribution. The critical value of the test statistic is 9.2 on a 1% level of

significance and is exceeded by the corresponding empirical values of the feeder pig prices (55.4), the pig prices (55.1) and the hog finishing margin (23.5). Thus the test results provide evidence that all distributions are fat tailed and justify the estimation of an extreme value distribution.

Figure 3: QQ-plots for feeder pigs, finished hogs, hog finishing margin



Application of the estimation procedure presented in section 3.2 is straightforward in principle, but the treatment of the hog finishing margin deserves a further comment. Two stochastic variables, the finished hog prices and the feeder pig prices, are involved in this case. Thus, the question arises how the EVT, which is designed for the estimation of univariate distributions, can

be adapted. Danielsson and De Vries (2000) describe two different approaches to apply EVT to a portfolio, namely post fitting and presampling. Post fitting is similar to Historical Simulation insofar as the returns of the different portfolio components are aggregated to give a univariate series of portfolio returns to which EVT can be applied. Correlations need not be estimated explicitly, but are implicitly assumed to be constant. Presampling, in contrast, is a multivariate approach. A tail estimation is carried out for each portfolio component and samples are drawn from the fitted distributions. A vector of portfolio returns is then calculated taking into account the sample covariance. Obviously post fitting is computationally much simpler and is preferred used here.

In order to motivate the aforementioned bootstrap procedure to determine the sample fraction for the tail estimation, we present Hill-estimators based on different values of k (figure 4). Apparently the result strongly depends on the number of extreme values which are included into the estimation. The extreme value distributions depicted in figure 5 are already based on the optimized number of extreme values. The respective figures are 6 for the feeder pig prices, 9 for the finished pig prices, and 3 for the finishing margin. The results of VCM and HS are also depicted for comparative purposes. The estimated tail indices of the extreme value distributions for the 1-week differences of feeder pig prices and pig prices are 5.37 and 4.08, respectively. Due to the positive correlation of the price changes of finished hogs and feeder pigs, the fluctuations of the hog finishing margin are smaller than those of the two single series. This is reflected by a relatively large tail index of 7.23 and corresponds also to the previous QQ-plots.

Figure 4: Tail estimators for different sample fractions

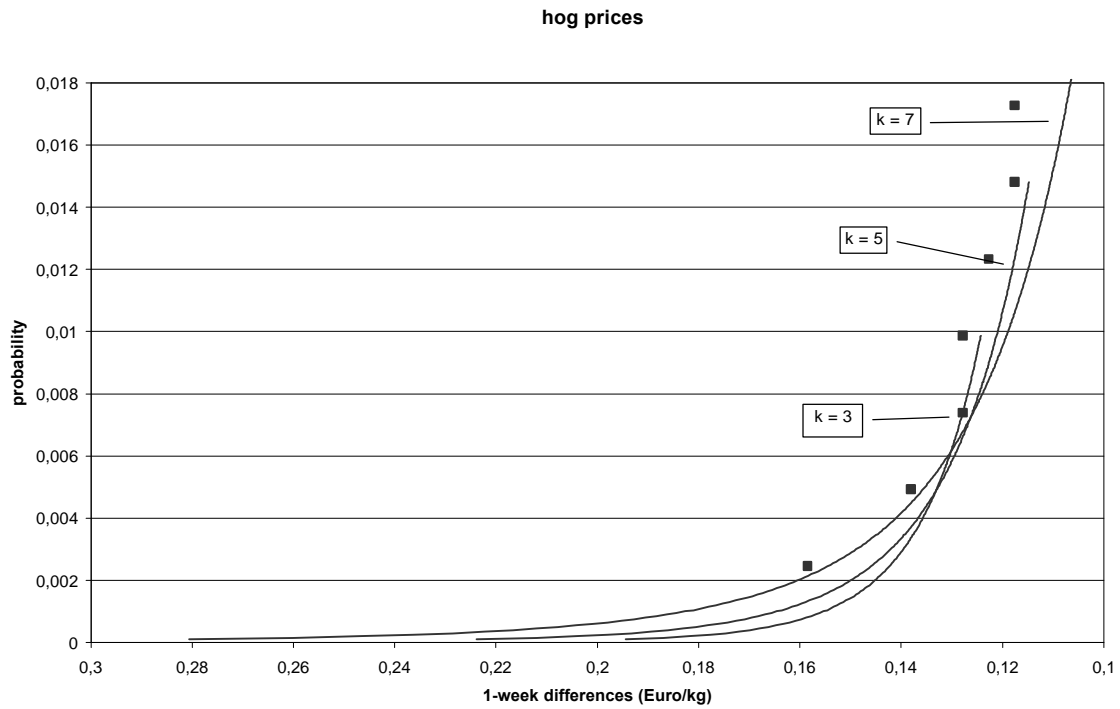
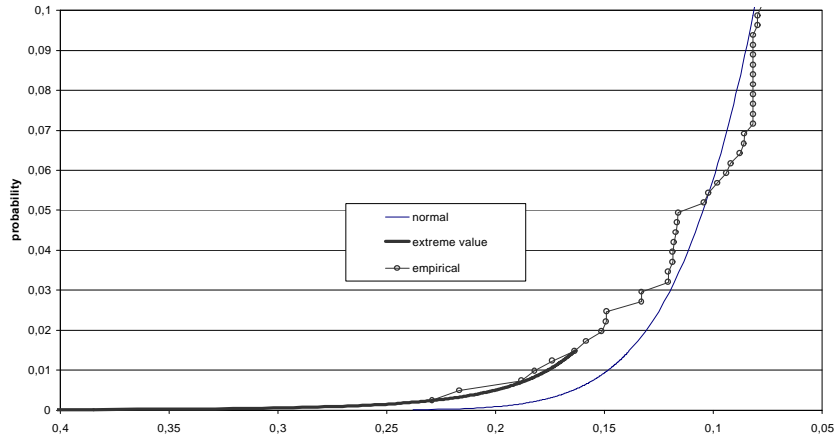
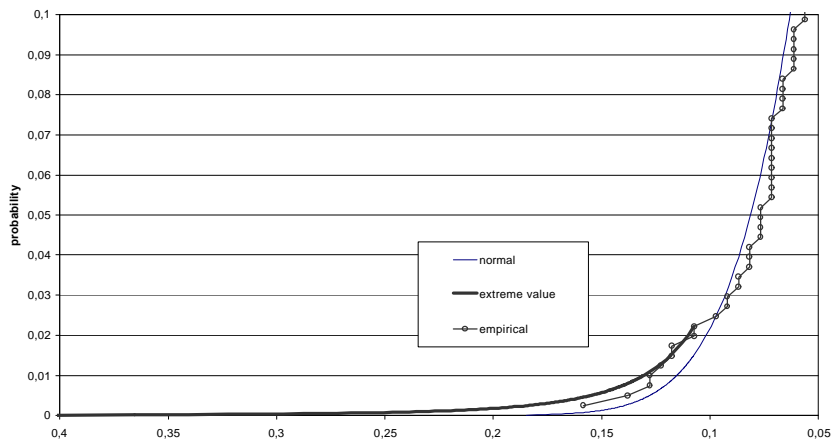


Figure 5: Normal distribution, empirical distribution and extreme value distribution

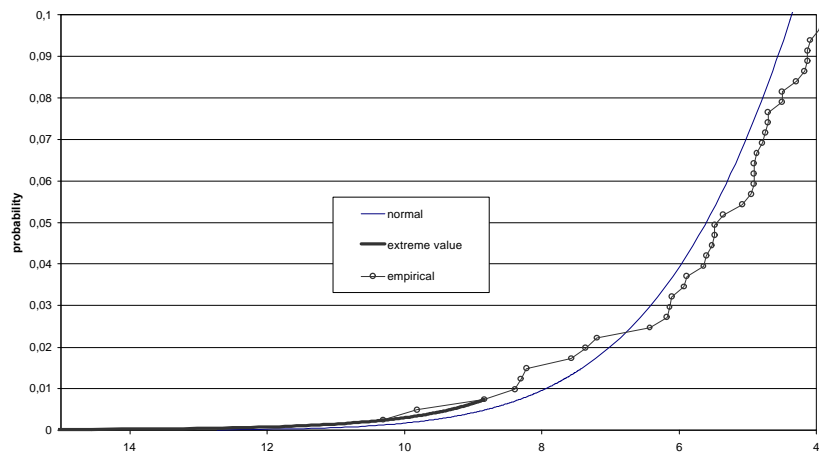
a) feeder pig prices (1-week-differences, Euro/kg)



b) hog prices (1-week-differences, Euro/kg)



c) hog finishing margin (1-week-differences, Euro/hog)



The last step consists of extrapolating the 1-week VaRs derived from figure 5 to the target horizon of 12 weeks. In the case of HS and VCM this is done with the square-root-rule, i.e. via multiplication with the factor 3.464. In contrast, the quantiles of the extreme value distribution are projected with the alpha-root-rule, i.e. using the respective tail indices $\hat{\alpha}$. Table 2 contains the results for different confidence levels. To allow a better comparison, the values of the extreme value distributions are also depicted for a 95% confidence level. According to the proposal of Danielsson and De Vries (2000) these values should be taken from HS, since they already lie to the right of the order statistics X_{k+1} .

Table 2: 1- and 12-week VaRs for the three time series and for different confidence levels (95%, 99%, 99.9%)

confidence level	feeder pigs			finished hogs			hog finishing margin		
	95.0%	99.0%	99.9%	95.0%	99.0%	99.9%	95.0%	99.0%	99.9%
	Euro			Euro			Euro		
EVT									
1 week	0.130	0.176	0.270	0.088	0.131	0.230	6.786	8.476	11.653
SE	0.012	0.005	0.085	0.006	0.009	0.058	1.034	0.203	1.862
12 week	0.207	0.280	0.429	0.162	0.240	0.422	9.567	11.950	16.429
HS									
1 week	0.104	0.182	-	0.077	0.128	-	5.358	8.303	-
SE	0.439	1.001	-	0.877	0.995	-	0.366	0.501	-
12 week	0.361	0.631	-	0.266	0.443	-	18.562	28.764	-
VCM									
1 week	0.105	0.148	0.197	0.081	0.115	0.153	5.607	7.947	10.571
SE	0.004	0.005	0.007	0.003	0.004	0.005	0.199	0.281	0.373
12 week	0.362	0.514	0.684	0.282	0.400	0.532	19.422	27.531	36.620

Compared to the EVT estimator, the VCM shows an underestimation of VaR for a short-term forecast. The underestimation, which increases with the confidence level, is a result of the assumed normality of the VCM and the actual leptokurtosis of the distributions. The 1-week VaR of the VCM for the feeder pigs, finished hogs, and hog finishing margin amounts to 0.197, 0.153, and 10.571 Euro at the 99.9% level. That means, the probability that the price for feeder pigs drops from its current level more than 0.197 Euros (19.7 Cents) is less than 0.1%. The

respective figures of the EVT are 0.27, 0.230, and 11.653 Euro. The differences should be related to the average prices (margin) of 1.938, 1.399 and 73.192 Euro.

HS and EVT differ only slightly at the 99% level. That means the distribution functions of the EVT and the HS intersect in that region (see figure 5). The VaR of the feeder pigs derived from HS is 0.182 Euro. This value is even higher than the corresponding EVT forecast which is 0.176 Euro. At the 99.9% level, quantiles can not be determined with HS, because losses of this size did not occur during the observation period.

Things appear completely different for the 12-week VaRs. HS and the VCM overestimate the medium-term VaRs relative to EVT. For example, the 95% quantile for the feeder pigs, finished hogs and hog finishing margin derived from the extreme value distribution amounts to 0.207, 0.162, and 9.567 Euro respectively, while the VCM display values of 0.362, 0.282, and 19.422 Euro, respectively. The corresponding values of the HS are 0.361, 0.266, and 18.562 Euro, respectively. Obviously, the short-term underestimation of the VaRs by HS and VCM is overcompensated by a too conservative time scaling via the square-root-rule.⁷ This bias becomes larger with increasing time horizon.

Table 2 further reports asymptotic standard errors of the estimated quantiles. The VCM seemingly shows the smallest estimation error. Some caution is necessary when interpreting the figures. The standard error (SE) of the VCM is calculated according to:

$$SE(\hat{x}_p) = \mathbf{s} (2n)^{-1/2} c_p \quad (20)$$

where \hat{x}_p denotes the estimated p -quantile, c_p is the p -quantile of the standard normal distribution and n denotes the numbers of observations. However, using (20) is only correct in case of normally distributed random variables. Since the normality assumption was rejected by the data, the displayed standard errors are incorrect as well. Calculation of the standard errors of the HS and the EVT is based on the expressions given in Jorion (1998, p. 99) and Danielsson and de Vries (1997). The figures in table 2 highlight the aforementioned pitfall of HS when it comes to an estimation of extreme quantiles. The standard errors of HS are relatively large for the given sample size of 405 observations. In this respect EVT offers a better alternative.

⁷ Mc Neil and Frey (2000) criticize the forecast applied here that essentially replaces the square-root-rule by an alpha-root-rule. They favor a two-stage procedure, which considers conditional heteroscedasticity in a first stage via GARCH estimation, and applies EVT to the residuals of the conditional estimation model in a second stage.

Usually some kind of validation is conducted subsequent to the VaR estimation. An out-of-sample prediction (backtesting) is a widespread validation technique. For that purpose the sample period is divided into an estimation period and a forecast period. Comparing theoretically expected and actually observed VaR violations occurring within the forecast period allows to validate competing models statistically (cf. Kupiec, 1995). However, such a validation is not possible in this application due to the relatively short observation period of the price series. An overshoot of a 99%-VaR would occur only once during 100 periods. In our application, such an event is expected to happen once within 100·12 weeks, i.e. once within 23 years. The impossibility of a model validation is not specific to our application. Rather it is an inevitable consequence of switching from a short-term to a long-term forecast horizon. Things are complicated by the fact that an EVT estimation requires excessive data.

5 Discussion and conclusions

The previous section exemplifies that the EVT can be applied to problems in agribusiness. For these specific agribusiness applications we found that:

1. Short-term VaR is underestimated in particular by the VCM when the return distributions are leptokurtic.
2. Using the alpha-root-rule instead of the common square-root-rule leads to a substantially smaller VaR for longer forecast horizons.
3. The accuracy of the estimation (expressed by asymptotic standard errors) increases compared to HS.

In considering the validity and the economic implications of these findings, as mentioned above, we have no direct statistical proof that EVT is superior to VCM and HS. Our assessment is rather based on theoretical arguments, namely the inappropriateness of VCM in case of nonnormal distributions and the statistical weakness of HS in tail estimation. Two things interfere with the generalization of our results. First, we cannot ensure that we have identified the best benchmark for the comparison of EVT and traditional VaR methods. In particular, the VCM was based on a rather simple volatility estimator, namely a long-run historical average. Manfredo and Leuthold (2001) consider several other estimators, amongst them exponentially weighted averages and implied volatilities. Second, it should be recalled that the EVT estimator is just an approximation for an unknown distribution. The quality of this approximation improves the more one moves towards the tails of the distribution, but it is impossible to specify a definite quantile where EVT becomes superior. As a rule of thumb some authors state that EVT should be used for estimating

quantiles greater or equal than 99% (cf. Danielsson, Hartmann, and de Vries, 1998). Hence we suggest further simulation experiments to learn more about the performance and the statistical properties of the different methods.

Another issue is the economic value of the information which is displayed by alternative VaR estimators. Opposed to financial institutions where VaR determines minimum capital requirements via the Basle accord⁸, the implications of a VaR forecast for risk management are less clear in non financial institutions such as farms. In our application VaR captures the maximal drop of important cash flow determinants (i.e. output and input prices). Changes of these market factors directly translate into changes of farm revenues. However, farm revenues depend on several other factors and therefore it is difficult to draw conclusions about the liquidity of the farm. For example, farms may have other cash generating activities than solely hog production. Cash outflows like debt service or wage payments will also differ between farms. Moreover, it seems necessary to include liquidity reserves into the analysis that have been generated during periods of high prices. Accordingly, the reported VaR forecasts should be embedded in a more comprehensive cash flow budget of the farm. Despite the difficulties in interpreting the VaR forecasts, we believe that it is important neither to overestimate nor to underestimate these values. For a given risk attitude, an overestimation of VaR will induce costly but unnecessary measures of risk reduction, e.g. holding excessive cash reserves. Reporting too high VaR values may also deteriorate the bargaining position of farmers who ask for debt capital. These examples highlight the relevance of our finding, that traditional methods tend to overestimate long-term VaR forecasts in the non iid case.

Finally some disadvantages of EVT should be mentioned. One drawback is that EVT is basically designed for the analysis of univariate distributions. Hence the primary advantage of VaR of being able to consider many risky market factors and to model their joint stochastic structure in a bottom-up approach is eroded. Another disadvantage is the increase of the computational burden of EVT compared to VCM or to HS. The reason for this is not the tail estimation itself, but the bootstrap procedure, which turned out to be necessary for the determination of an optimal sample fraction. However, this disadvantage is weakened, since a tail index estimation will be executed less frequently compared with short-term financial applications, where a permanent updating of VaR forecasts is required when new price information becomes available.

To summarize, the benefits of displaying extreme quantiles depend on the specific problem. Apparently, the informational needs concerning risk differ largely e.g. between a hog producer, a broker trading with hog futures, and an insurance company insuring against animal diseases. In some cases the inclusion of additional sources of risk seems more important than to push the confidence level of VaR from 95% to 99.9%. For example, the production risks emanating from foot and mouth disease or BSE for a individual producer, are not echoed by aggregated market prices. However, *if* a calculation of extreme quantiles (e.g. 99% or higher) appears desirable then EVT should be used as a supplement. Additional cost of computation are overruled by a higher accuracy of the tail estimates as well as by significant differences in the temporal aggregation of VaR whenever leptokurtic distributions are involved.

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⁸ The linkage between VaR and capital charges of banks due to the Basle accord is discussed in Jorion (1997, ch. 3).

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7 Appendix

Bootstrap method for the determination of the sampling fraction k/n

A solution to the determination of the sampling fraction k/n is given by means of a bootstrap method, as suggested by Danielsson et al. (2001). First resamples $N_{n_1}^* = \{X_{n_1}^*, \dots, X_{n_1}^*\}$ of predetermined size $n_1 < n$ are drawn from the data set $N_n = \{X_1, \dots, X_n\}$ with replacement. For any k_1 the asymptotic mean square error (AMSE) $Q(n_1, k_1)$ is calculated:

$$Q(n_1, k_1) = E \left[\left(M_{n_1}^*(k_1) - 2(\mathbf{x}_{n_1}^*(k_1)) \right)^2 \mid \mathbf{N}_n \right] \text{ with} \quad (\text{A1})$$

$$M_{n_1}^*(k_1) = \frac{1}{k_1} \sum_{i=1}^{k_1} (\ln X_{n_1,i}^* - \ln X_{n_1,k_1+1}^*)^2 \text{ and} \quad (\text{A2})$$

$$\mathbf{x}_{n_1}^*(k_1) = \frac{1}{k_1} \sum_{i=1}^{k_1} \ln X_{n_1,i}^* - \ln X_{n_1,k_1+1}^* \quad (\text{A3})$$

Then find $k_{1,0}^*(n_1)$, i.e. the value of k_1 , which minimizes the AMSE (A1):

$$k_{1,0}^*(n_1) = \operatorname{argmin} Q(n_1, k_1) \quad (\text{A4})$$

A second step completely analogous to the first one but with a smaller sample size $n_2 = (n_1)^2/n$ yields $k_{2,0}^*(n_2)$. Next calculate

$$\hat{k}_0(n) = \frac{(k_{1,0}^*(n_1))^2}{k_{2,0}^*(n_2)} \left(\frac{(\ln k_{1,0}^*(n_1))^2}{(2 \ln n_1 - \ln k_{1,0}^*(n_1))^2} \right)^{\frac{\ln n_1 - \ln k_{1,0}^*(n_1)}{\ln n_1}} \quad (\text{A5})$$

This allows to calculate the reciprocal tail index estimator:

$$\mathbf{x}_n \left(\hat{k}_0 \right) = \frac{1}{\hat{k}_0} \sum_{i=1}^{\hat{k}_0} \ln X_{n_1,i}^* - \ln X_{n_1,\hat{k}_0+1}^* \quad (\text{A6})$$

This estimation procedure for k depends on two parameters, the number of bootstrap resamples, l , and the sample size, n_1 . The number of resamples is in general determined by the available computational facilities. The application presented in section 4 utilizes 10,000 repetitions giving

very stable results. Evaluation and optimization of n_1 , necessitates a further step. Calculate the ratio

$$R(n_1) = \frac{(Q(n_1, k_{1,0}^*))^2}{Q(n_2, k_{2,0}^*)} \quad (\text{A7})$$

and determine $n_1^* = \text{argmin} R(n_1)$ numerically. If n_1^* differs from the initial choice n_1 , the previous steps should be repeated. Remember that the quantile estimates derived from EVT are only valid for the tails of the profit-and-loss distribution. To allow inferences about quantiles in the interior of the distribution, Danielsson and de Vries (2000) propose to link the tail estimator with the empirical distribution function at the threshold X_{k+1} . Thus the particular advantages of the EVT and the HS are combined.